Notes / Assumption

$Pa = \frac{N}{m^2}$ $N = kg\frac{m}{s^2}$

Tank @ 50°c or 323.15 K

$C_{v_{CO2}} = 28.96$ J/mol·K

Supercritical state @ 73.8 bar, 1070.37 Psi
Tank
7.38 Mpa

Incompressible Fluid

Tank Volume 52.11 in^2 = 0.000853 m^3

$\rho = 200\text{-}400 \frac{kg}{m^3}$ @ 75 bar

mol mass = 0.01201 kg/mol

Redlich Kwong Eq of State

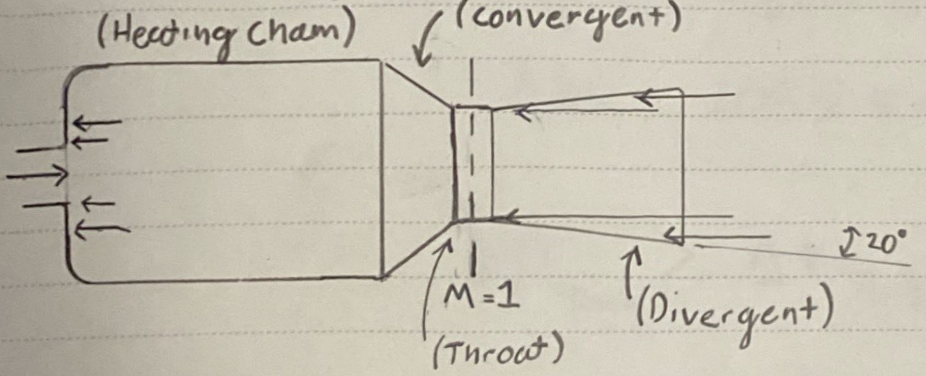
$$P = \frac{RT}{V-b'} - \frac{a'}{(\sqrt{T})v(v+b')} \qquad V_c = \frac{RT_c Z_c}{P_c}$$

V = molar
P = Pressure
T = Temp
R = gas constant

$$a' = \left(\frac{0.4278}{Z_c}\right) R V_c T_c^{3/2} \qquad b' = \left(\frac{0.0867}{Z_c}\right) V_c$$

$T_c, Z_c, P_c \}$ All tables

Rocket Nozzle



Specific impulse
(Efficiency)

$I_s = V_2 / g$

Conservation of Momentum
(time = 0)

$m_i v_i = 0 \Rightarrow m_\gamma V_\gamma = m_p V_p$

$$m_\gamma \underbrace{\frac{dv_\gamma}{dt}} = -\underbrace{\frac{dm}{dt}} V_p$$

(note)

$F = \dot{m} V_2 + (P_2 - P_3) A_2$

F=ma (Ideal Nozzel Expansion)

$m_\gamma a_\gamma = -\dot{m} V_p$

$F = \dot{m} V_p$

(Diagram)

(System Energy Equation)

$\Delta Q = 0$

ΣE (closed system, $\Delta M = 0$)

Tank Volume

$V = 0.000853\ m^3$

$$(\overset{0}{\cancel{w_{in} - w_{out}}}) + (\overset{0}{\cancel{Q_{in} - Q_{out}}}) = \overset{0}{\cancel{\Delta U}} + \Delta KE + \Delta PE$$

$P = 7.38 \times 10^6\ Pa$

$Pa = \frac{N}{m^2}$

$N = kg\ \frac{m}{s^2}$

$$\Sigma E = \Delta KE + \Delta PE$$

(TANK P.E.)

$$\Delta PE = P(V)$$

$$\Delta PE = (7.38 \times 10^6)(N/m^2)(0.000853)(m^3)$$

$$\Delta PE = 6295.14\ N \cdot m$$

(Body Falling)

$$\Delta KE = (5kg)(9.8\ m/s^2)(3m) + \frac{1}{2}(5kg)(5\frac{m}{s})$$

$$= 147\ kg\ \frac{m^2}{s^2} + 62.5\ kg\ m^2/s^2$$

$$\Delta KE = 209.5\ N\ m$$

$$\Delta PE_{Tank} > \Delta KE_{Body}$$

(Bernoulli's Equation)

$\Delta P = 1070 Psi - 14.7 Ps$

$= 1055.3 Psi$

$= 7.38 * 10^6 Pa$

$V = \sqrt{\frac{2\Delta P}{\rho}}$

$V = \boxed{135.83 \, m/s}$

$\rho = 800 \, kg/m^3$

$V = \, m/s$

(Mass Flow)
From TANK

$V = 135.83 \, m/s$

$\rho = 800 \, kg/m^3$

$20oz = 0.5669 \, kg$

$\emptyset = 0.25 = 6.37 \, mm$

$A = 0.0000 \, 3167 \, m^2$

$\dot{m} = \rho A V$

$3/16 = 0.004762 \, m$

$A = 0.00001781 \, m^2$

$\emptyset = 0.25$

$1/8 = 0.003175 \, m$

$A = 0.000007917 \, m^2$

$\dot{m} = (400 \, kg/m^3)(0.00003167 m^2)(135.83 \, \frac{m}{s})$

$\dot{m} = 1.720 \, kg/s$

Time

$\frac{m}{\dot{m}} = t$

$\frac{0.5669 \, kg}{1.720} \left(\frac{s}{kg}\right) = 0.3294 \, s$

$\varnothing = 3/16$

$$\dot{m} = (1400\ kg/m^3)(0.00001781\ m^2)(135.83\ m/s)$$

$$\dot{m} = 0.9676\ kg/s$$

Time

$$\frac{0.5669\ kg}{0.9676}\left(\frac{s}{kg}\right) = 0.5858\ s$$

$\varnothing = 1/8$

$$\dot{m} = (1400\ kg/m^3)(0.000007917\ m^2)(135.83\ m/s)$$

$$\dot{m} = 0.4301\ kg/s$$

$$\frac{0.5669\ kg}{0.4301}\ \frac{s}{kg} = 1.3180\ s$$

$\varnothing_{Act} = 0.340\ in$ A = $0.00005858\ m^2$

$$\dot{m} = (1400\ kg/m^3)(0.00005858\ m^2)(135.83\ m/s)$$

$$\dot{m} = 3.1827\ kg/s$$

(Flow is divid into 3 Flow Paths)

$\dot{m}$ $\frac{\dot{m}_{total}}{}$ $\dot{m}_{tot} = \dot{m}_1 + \dot{m}_2 + \dot{m}_3$

(Mass Flow)

Frist Face $A_{F1} = 0.00003365 m^2$ $\dot{m}_{FF} = 1.0609\ kg/s$

$$\dot{m}_{FF} = \rho A V$$

$$1.0609\ kg/s = (400\ kg/m^3)(0.00003365 m^2)(V)$$

$$V_{FF} = 78.818\ m/s$$

Heater Tube Divid by two Flow

$A = 0.00001140\ m^2$

$$\dot{m}_{HT} = \frac{\dot{m}_{FF}}{2} = 0.530$$

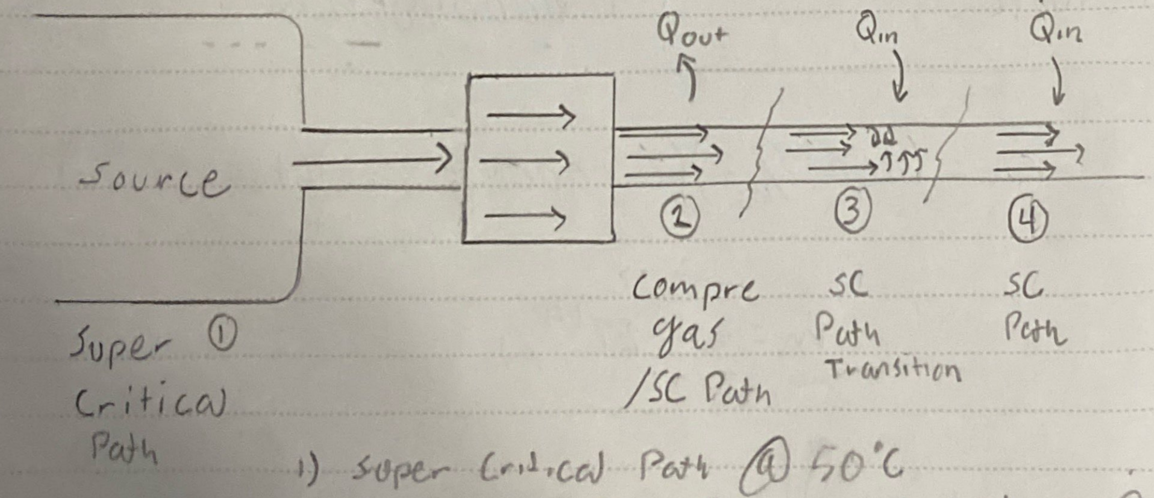
$$\dot{m}_{HT} = \rho A V$$

$$\dot{m}_{HT} = \dot{m}_1 + \dot{m}_2$$

$$0.5304\ \frac{kg}{s} = (400\ kg/m^2)(0.00001140 m^2)\ V$$

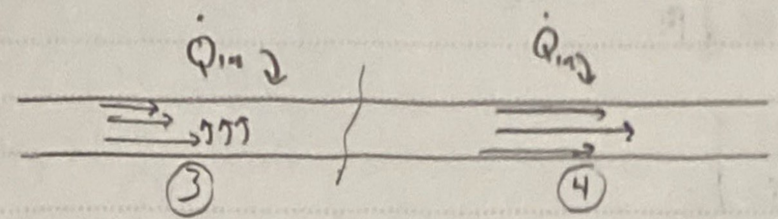
$$116.326\ m/s = V$$

Theoretical Prediction



1) Super Critical Path @ 50°C
2) Compre/SC Path Because of cooling $\dot{Q}_{out}$
3) Transition zone SC $\dot{Q}_{in}$
4) SC Flow

Theoretical Prediction



(CO$_2$)

$C_v = 28.96 \text{ J/mol·K}$

mol mass $= 0.01201 \text{ kg/mol}$

$\dot{Q}_{total} = 2500 \text{ J/s}$

$\dot{m}_{HT} = 0.5304 \text{ kg/s}$

$$\overset{\text{Heat energy}}{\dot{Q}_{in}} = \underset{\text{mass Flow}}{\dot{m}}\ \overset{\text{heat capacity}}{C}\ \Delta t$$

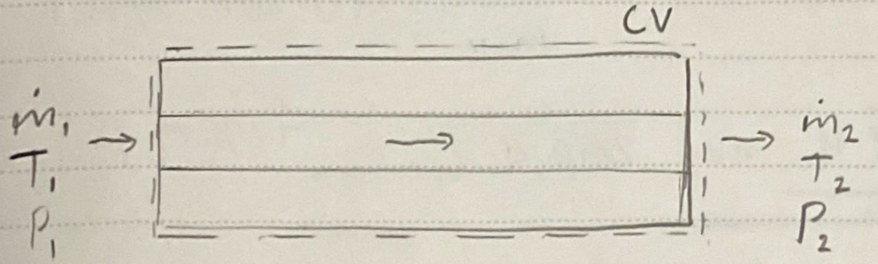
$$\dot{Q}_{\substack{per\\path}} = \frac{\dot{Q}_{total}}{6} = \frac{2500 \text{ J/s}}{6} = 416.6 \text{ J/s}$$

$$2500 \text{ J/s} = 44.163 \text{ mol/s} \, (28.96 \text{ J/mol·K}) \, \Delta t$$

$$\boxed{1.95 \text{ K} = \Delta t}$$

$H = H'(T) + \int (C_p \, dT) - (PV$

Heat Exchanger mobel



STEADY-STATE

$\dot{m}_1 = \dot{m}_2$

$\dot{m}_1 = 0.5304 \text{ kg/s}$

$T_1 = 325.15 \text{ K}$

$P_1 = 7.38 \text{ MPa}$

$\dot{Q}_{in} = 416.6 \text{ J/s}$

$$\frac{dE}{dt} = 0 = \dot{Q}_{in} - \cancelto{0}{\dot{W}} + \dot{m}_1 \left(h_1 + \cancelto{0}{\frac{V_1^2}{2}} + \cancelto{0}{g z_1}\right) - \dot{m}_2 \left(h_2 + \cancelto{0}{\frac{V_1^2}{2}} + \cancelto{0}{g z_2}\right)$$

$$0 = \dot{Q}_{in} + \dot{m}_1 (h_1) - \dot{m}_2 (h_2)$$

H Enthalpy : 472800 J/kg

Enthalpy : 1863 J/kg·K

$416.6 \text{ J/s} = 0.5304 \text{ kg/s}$

$\dot{Q}_{in} = -\dot{m} (h_1 - h_2)$

$-785.444 \text{ J/kg} = 472800 \text{ J/kg} - h$

$h = 473585.4441 \text{ J/kg}$

Date:

Fudamental Thermodynam
Relationships

$$dH = T\,ds + V\,dP$$

$$\left.\frac{\partial H}{\partial s}\right|_P = T \qquad \left.\frac{\partial H}{\partial P}\right|_s = V$$

(Maxwell Relations)

$$\left.\frac{\partial T}{\partial P}\right|_s = \left.\frac{\partial V}{\partial s}\right|_P$$

$$dU = T\,ds - P\,dv$$
$$dH = T\,ds + V\,dP$$

$F(x,y)$

$$dF = \left.\frac{\partial F}{\partial x}\right|_y dx + \left.\frac{\partial F}{\partial y}\right|_x dy$$

Thermodynamic Quantiti

$$C_v = \left.\frac{\partial U}{\partial T}\right|_V$$

Heat Exchanger model

$k_2 = 473589.444 \; J/kg \times \frac{mol}{mass}$ $\qquad \dot{m} = 0.5304 \; kg/s$

$$\Delta T = \frac{5687.761 \; J/mol}{C_v} \; \frac{mol \cdot K}{J}$$

$C_v = 28.96 \; J/mol \cdot K$

$\Delta T = 196.40 \; K$

$\frac{mol}{mass} = 0.01201 \; kg/mol$

$\underline{T_2 = 521.55 \; K}$ or 248.4 C, 479.12 F

(RK Eq of State)

$$P = \frac{RT}{V - b'} - \frac{a'}{(\sqrt{T})\, V(V + b')}$$

$$V_c = \frac{R T_c Z_c}{P_c}$$

$$a' = \left(\frac{0.4278}{Z_c}\right) R V_c T_c^{3/2}$$

$$b' = \left(\frac{0.0867}{Z_c}\right) V_c$$

$$\dot{m}_2 = 0.5304 \frac{kg}{s}$$

$$T_2 = 521.55k$$