

Date:

# (Energy Equation)

Tank Volume

$$52.11 \text{ in}^3 = 0.000853 \text{ m}^3$$

$$P = 900 \text{ PSI} = 62 \text{ bar}$$

$$6205500 \text{ Pa}$$

$$Pa = \frac{N}{m^2}$$

$$N = \frac{kg \cdot m}{s^2}$$

$$\Sigma E \text{ (closed system, } \Delta M = 0)$$

$$(\cancel{W_{in}} - \cancel{W_{out}}) + (\cancel{Q_{in}} - \cancel{Q_{out}}) = \cancel{\Delta U} + \Delta KE + \Delta PE$$

$$\Sigma E = \Delta KE + \Delta PE$$

$$\Delta PE = P(V) \quad \text{Tank P.E.}$$

$$\Delta PE = (6205500 \text{ (N/m}^2)) (0.000853 \text{ m}^3)$$

$$\Delta PE = 5293.29 \text{ N}\cdot\text{m}$$

## Body Falling

$$\Delta KE = (5 \text{ kg})(9.8 \text{ m/s}^2)(3 \text{ m}) + \frac{1}{2} (15 \text{ kg})(5 \text{ m/s})^2$$

$$\Delta KE = 147 \text{ kg}\cdot\text{m}^2/\text{s}^2 + 62.5 \text{ kg}\cdot\text{m}^2/\text{s}^2$$

$$\Delta KE = 209.5 \text{ N}\cdot\text{m}$$

$$\Delta PE_{\text{Tank}} > \Delta KE_{\text{Body}}$$

$$0 = 2.88 \text{ m} \times 3.2 \text{ g/m}$$

0.16 m

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# Redlich Kwong Eq of State

$$P = \frac{RT}{V-b'} - \frac{a'}{\sqrt{T} V (V+b')}$$

$V$  = molar Volume  $\frac{\text{Vol}}{\text{mol}}$   
 $P$  = Pressure  
 $T$  = Temp (Kelvin)  
 $R$  = gas constant

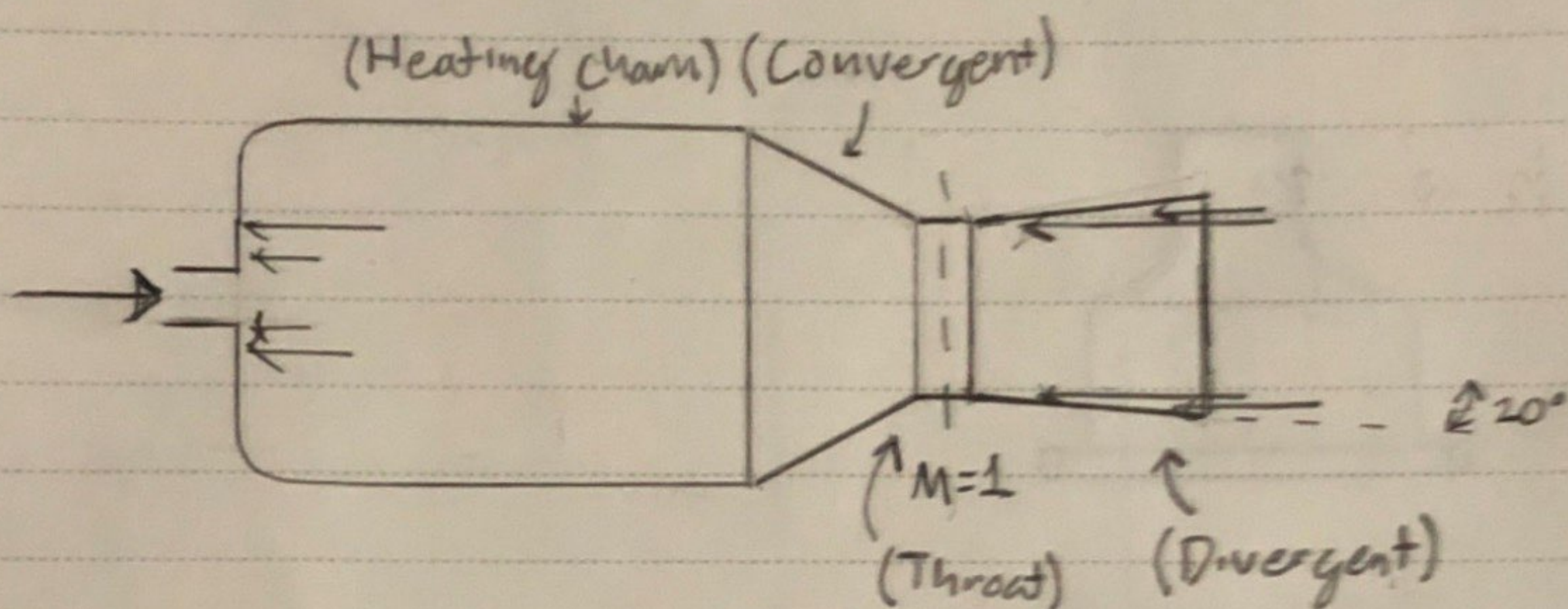
$$a' = \left( \frac{0.4278}{Z_c} \right) R V_c T_c^{3/2}$$

$T_c, Z_c, P_c$  } all tabulated

$$b' = \left( \frac{0.0867}{Z_c} \right) V_c$$

$$V_c = \frac{RT_c Z_c}{P_c}$$

## Rocket Nozzle



Specific (note)  
 impulse  
 (Efficiency)  
 $I_s = v_2 / g$

## Conservation of Momentum

(time = 0)

$$m_i v_i = 0 \Rightarrow m_y v_y = m_p v_p$$

$$m_y \frac{dv_y}{dt} = - \frac{dm}{dt} v_p$$

$\underbrace{\quad}_{\text{acceleration}} \quad \underbrace{\quad}_{\dot{m}}$

(note)

$$F = \dot{m} v_2 + (P_2 - P_3) A_2$$

$$F = ma$$

$$m_y a_y = - \dot{m} v_p$$

$$F = \dot{m} v_p$$

(Ideal Nozzle Expansion)

# (Bernoulli's Equation)

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$$P_1 + \frac{1}{2} \rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g z_2$$

$$850 \text{ psi} = 5.86 \text{ Mpa}$$

(TANK)

$$P_1 = 5.86 \text{ Mpa}$$

(ATM)

$$P_2 = 0.1013 \text{ Mpa}$$

$$= 101325 \text{ Pa}$$

$$5.86 \text{ Mpa} + (1.87 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.3 \text{ m}) = 0.1013 \text{ Mpa} + \frac{1}{2}(1.87 \text{ kg/m}^3)V_2^2$$

$$5.758 \text{ Mpa} + 5.4976 \text{ kg/m}^2 = \frac{1}{2}(1.87 \text{ kg/m}^3)V_2^2$$

$$11.5174 \text{ Mpa} + 10.9956 \text{ kg/m}^2 = 1.87 \text{ kg/m}^3 V_2^2$$

$$1 \text{ Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg m/s}^2}{\text{m}^2}$$

$$\frac{1}{1.87} \frac{\text{m}^3}{\text{kg}} (11.5174 \times 10^6 \frac{\text{kg m}}{\text{s}^2 \text{ m}^2} + 10.9956 \text{ kg/m}^2) = V_2^2$$

$$\rho = 1.87 \text{ kg/m}^3$$

@ 850 PSI

$$6159037.43 \frac{\text{m}^2}{\text{s}^2} + 5.88 \frac{\text{m}^2}{\text{s}^2}$$

$$6159043.31 \frac{\text{m}^2}{\text{s}^2} = V_2^2$$

$$\boxed{2481.74 \text{ m/s} = V_2}$$

$$\frac{\text{m}}{\text{s}} = \frac{\text{m}^2}{\text{m}^2 \text{ s}}$$

(time) note

$$2002 = 0.56 \text{ kg/s}$$

$$\dot{m} = \rho A V$$

$$D = 0.25 \text{ in} = 6.37 \text{ mm}$$

$$A = 0.00003167 \text{ m}^2$$

$$0.56 \text{ kg/s} = 3.85 \text{ s} \quad \boxed{\dot{m} = 0.1469 \text{ kg/s}}$$

$$\boxed{\text{Time } 3.85 \text{ s}}$$

Conservation of Momentum (without Nozzle)

$$m_y a_y = -\dot{m} V_p$$

$$m_y (9.8 \text{ m/s}^2) = -(0.1469 \text{ kg/s})(2481.74 \text{ m/s})$$

$$m_y (9.8 \text{ m/s}^2) = -364.75 \frac{\text{kg m}}{\text{s}^2}$$

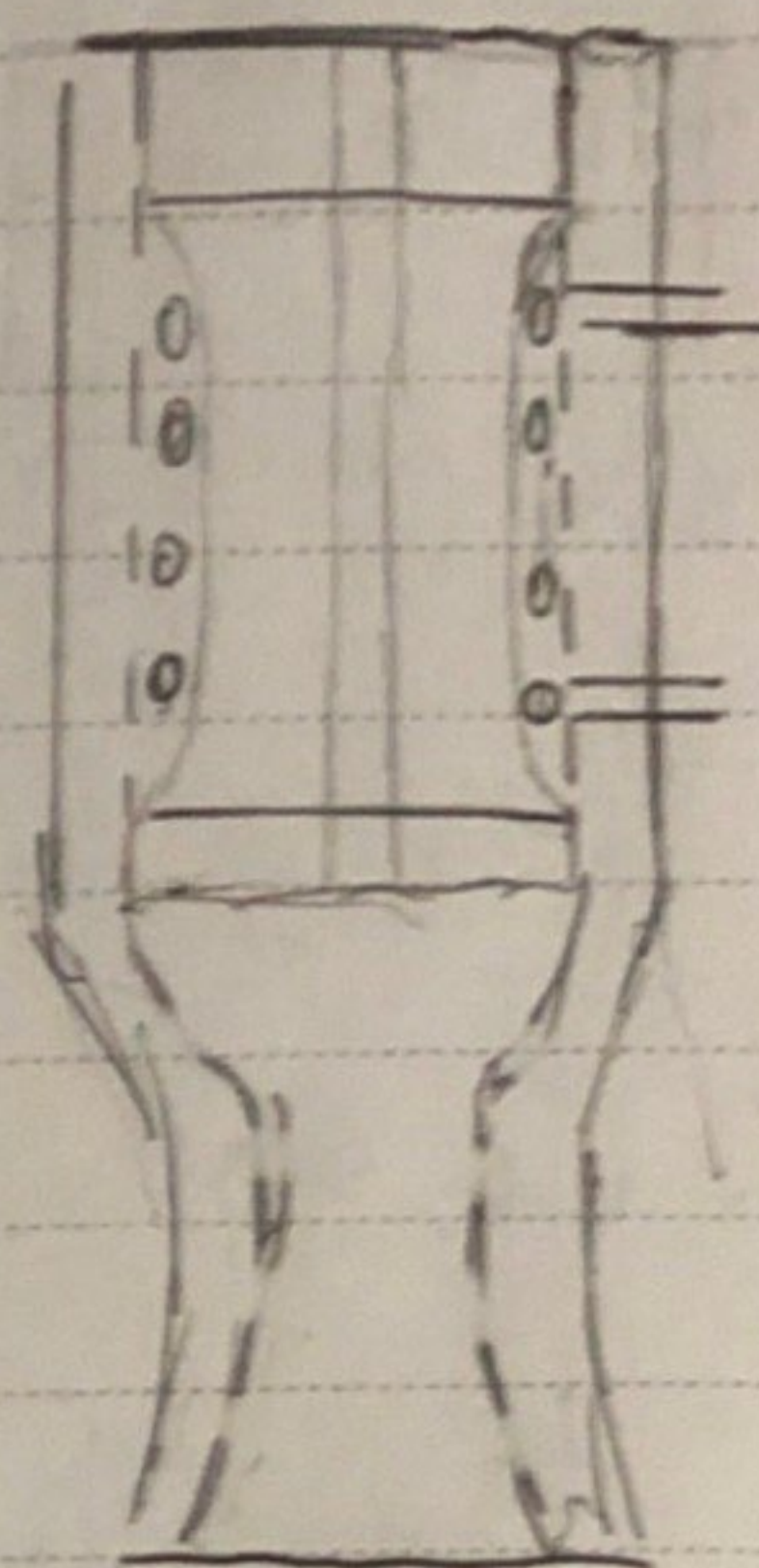
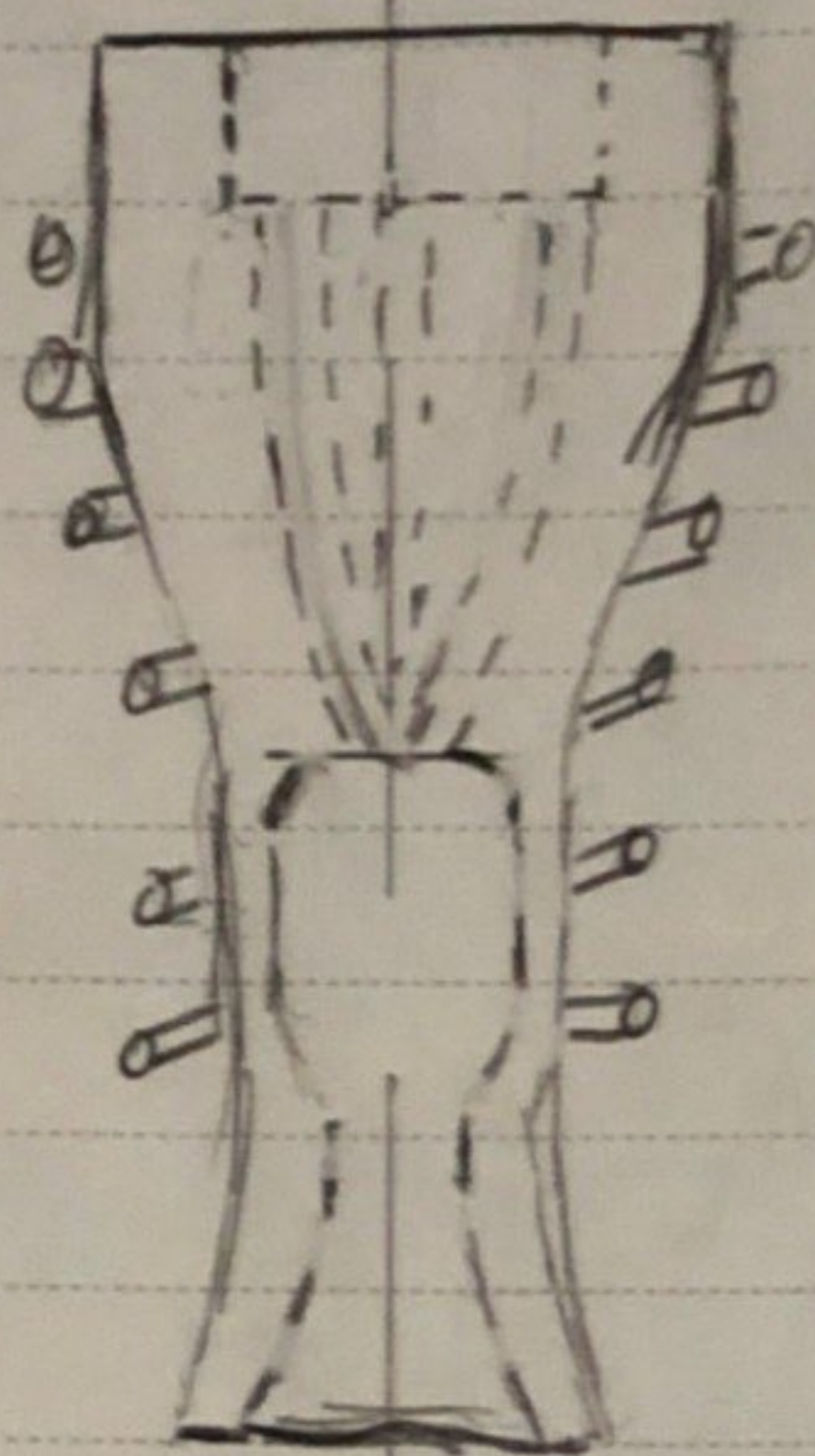
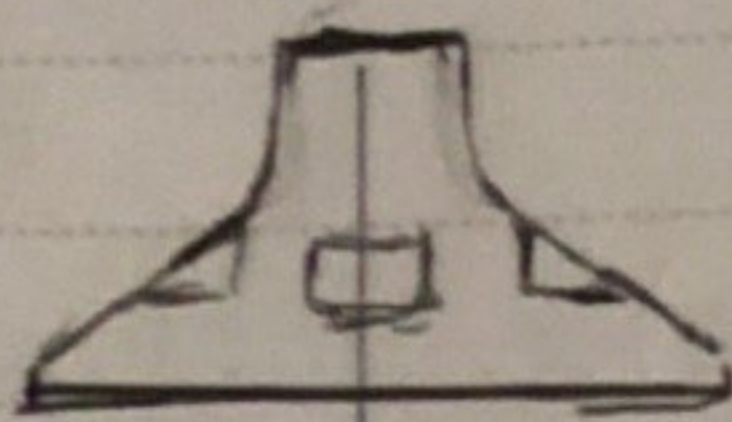
$$\boxed{m_y = 37.21 \text{ kg}}$$

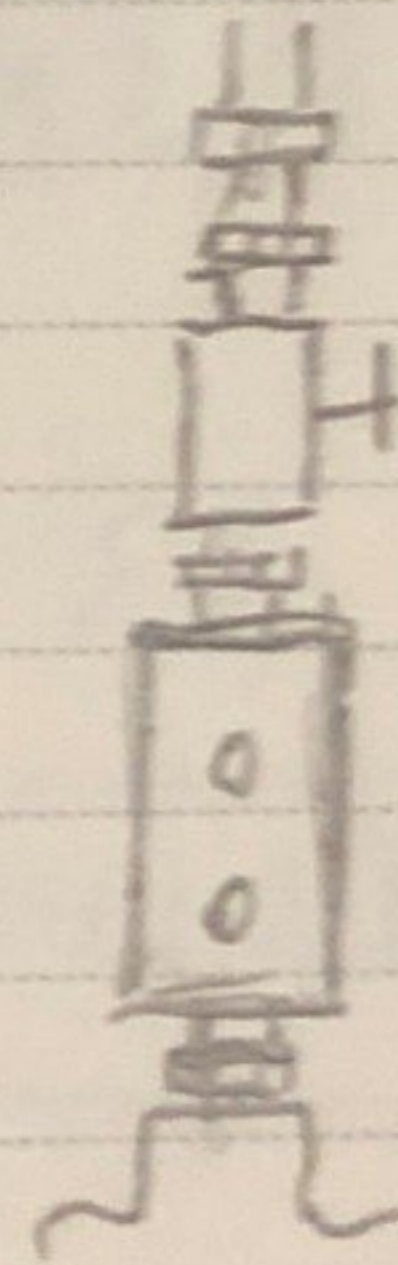
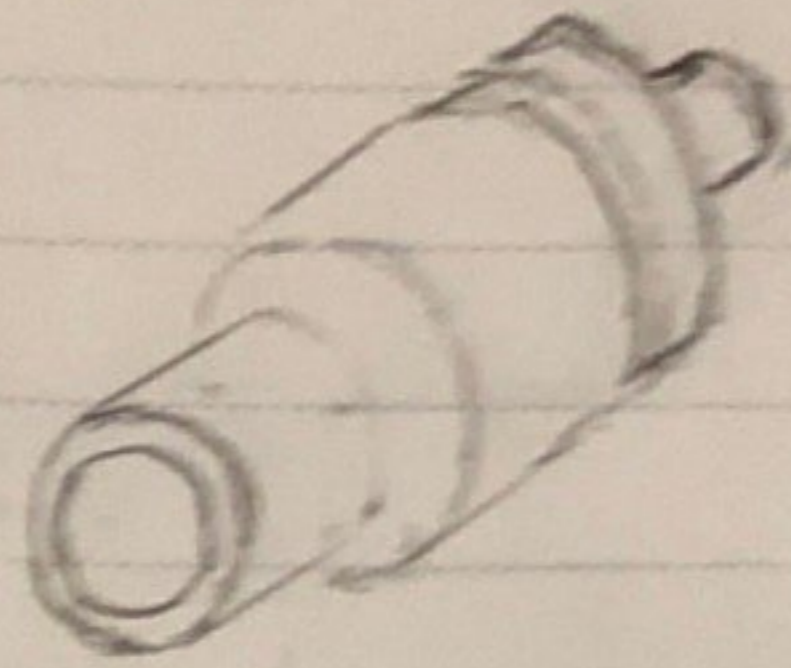
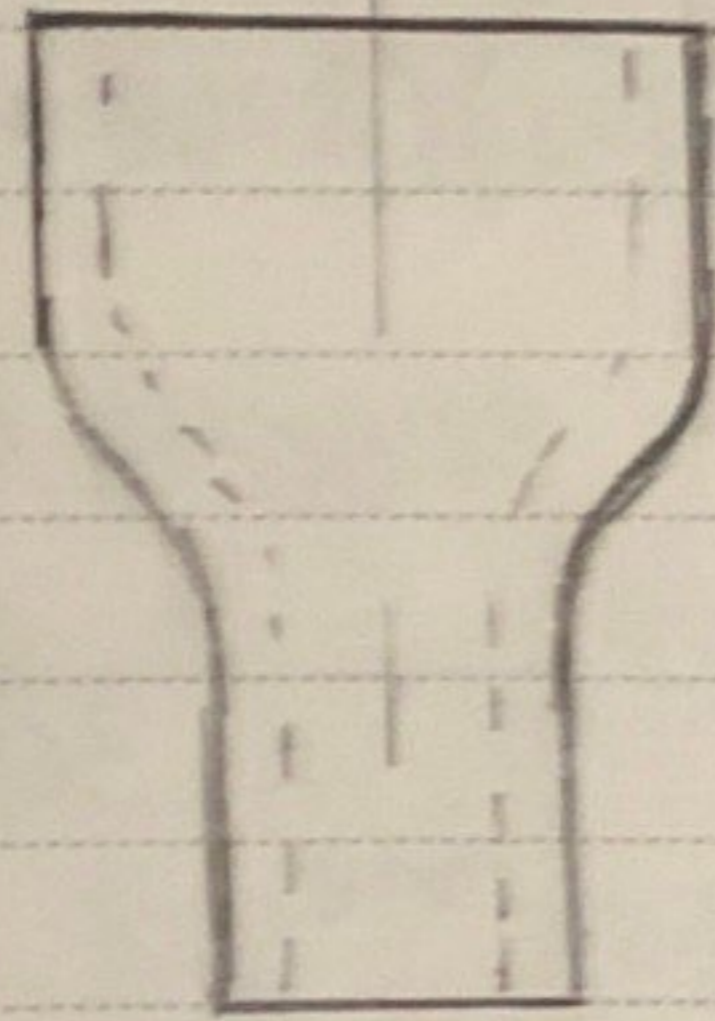
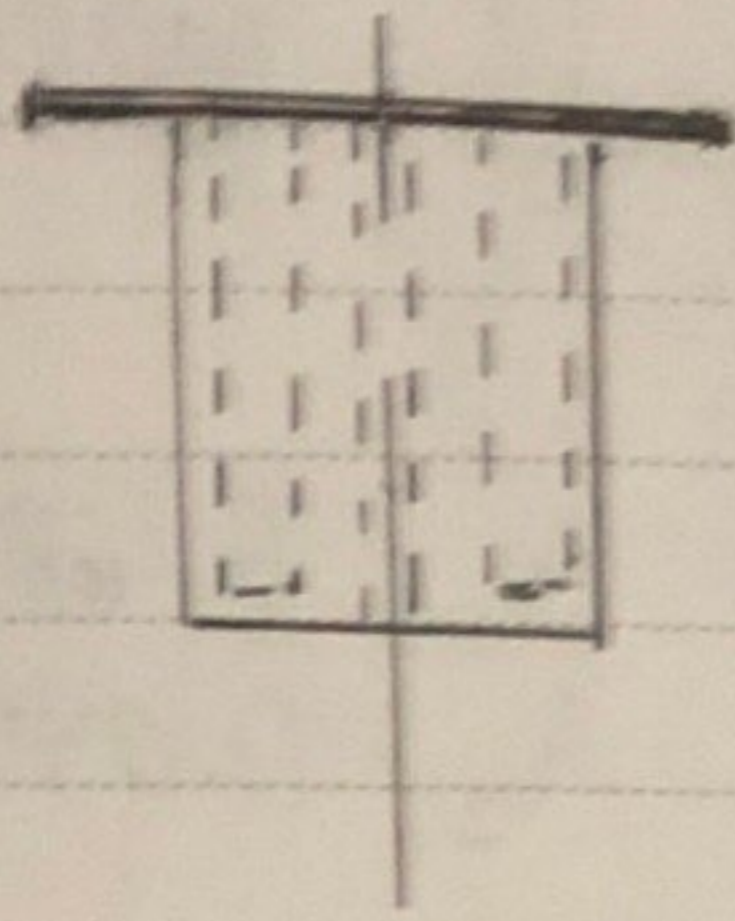
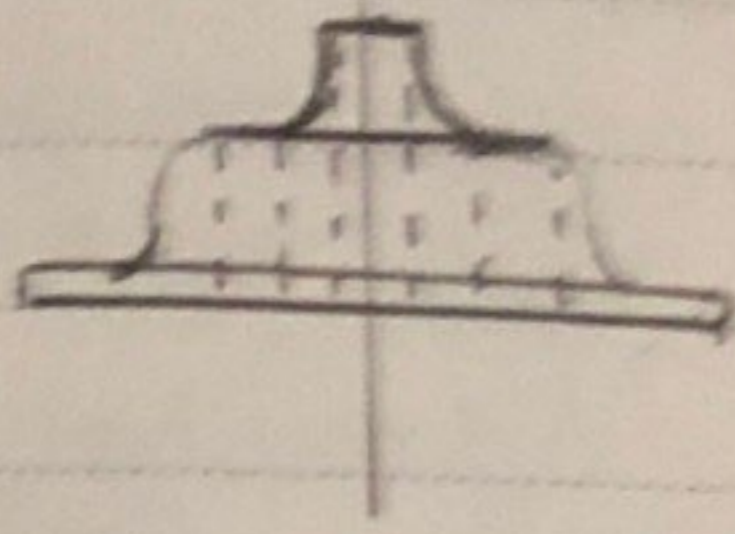
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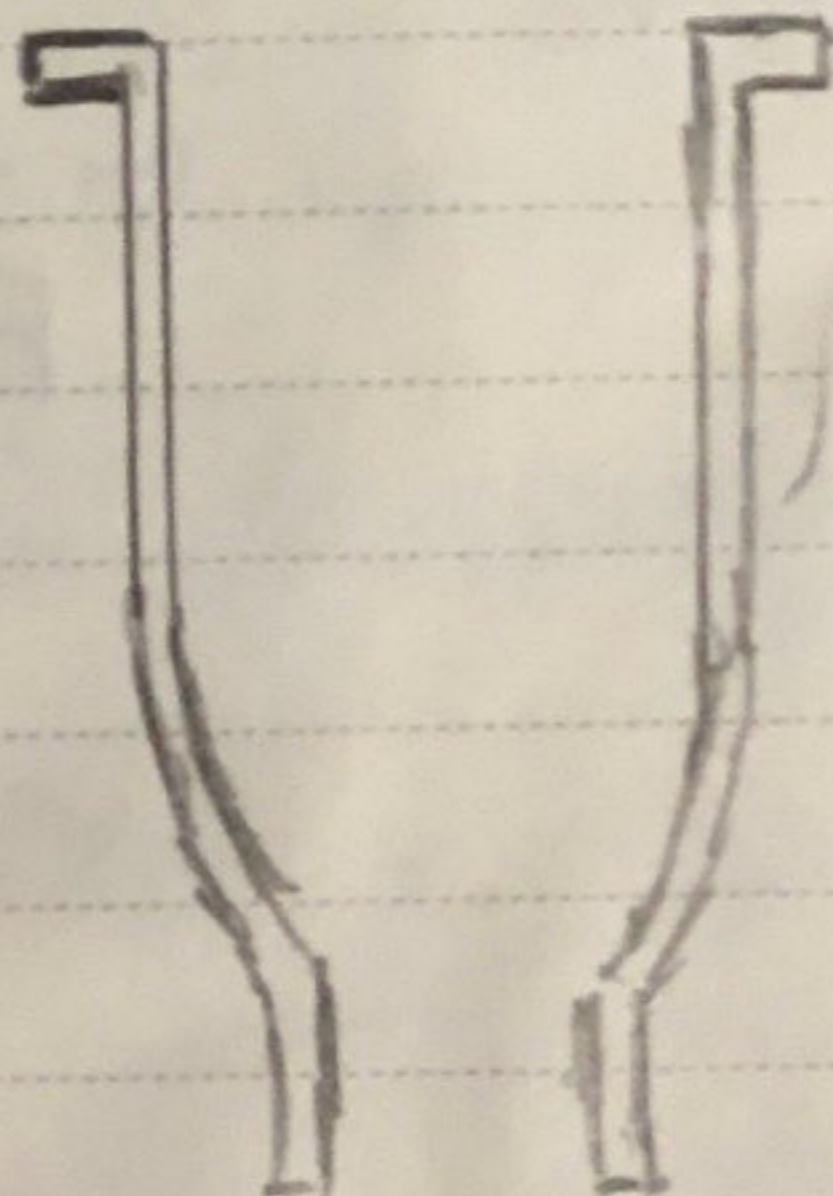
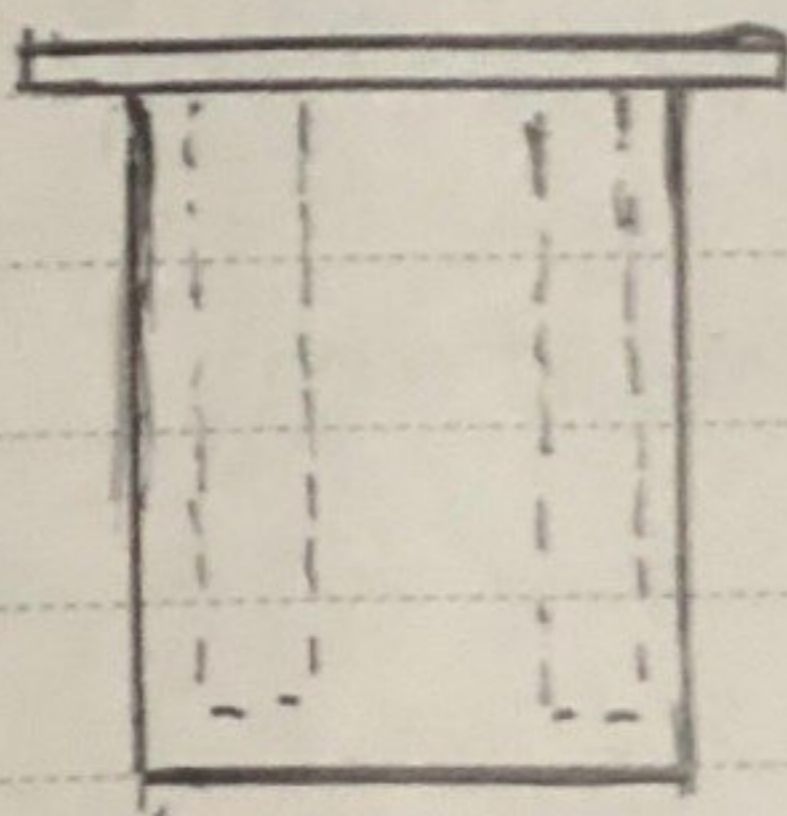
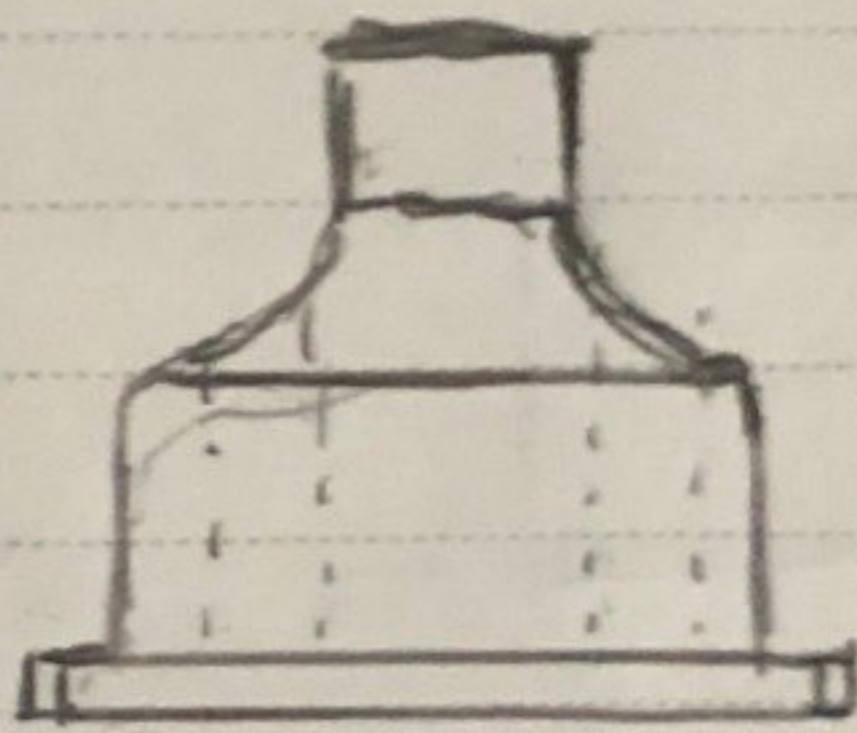
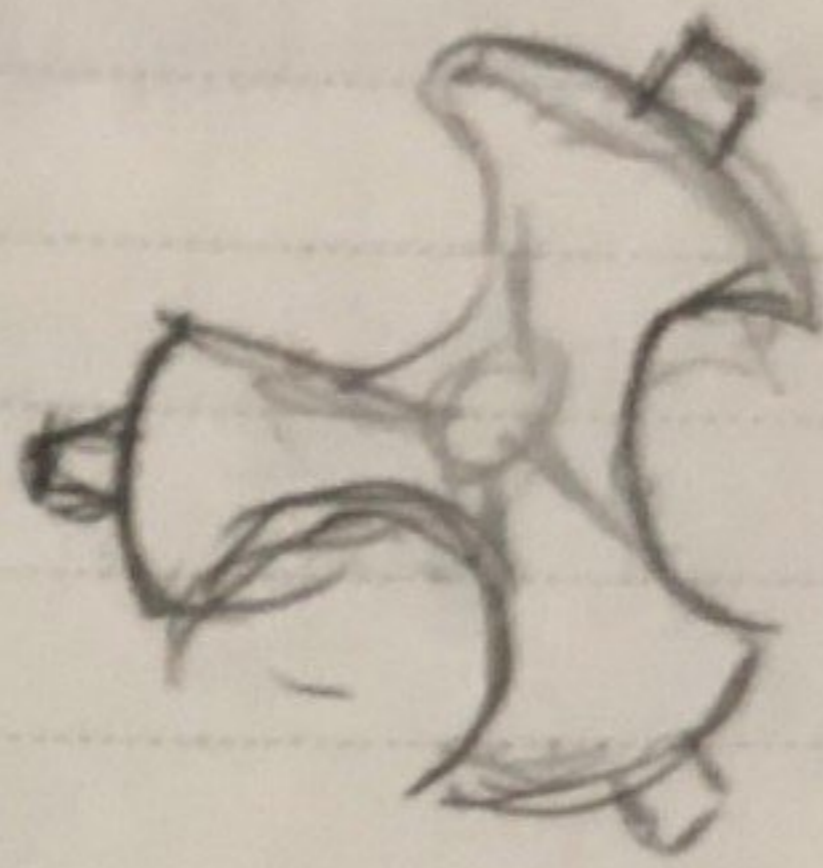
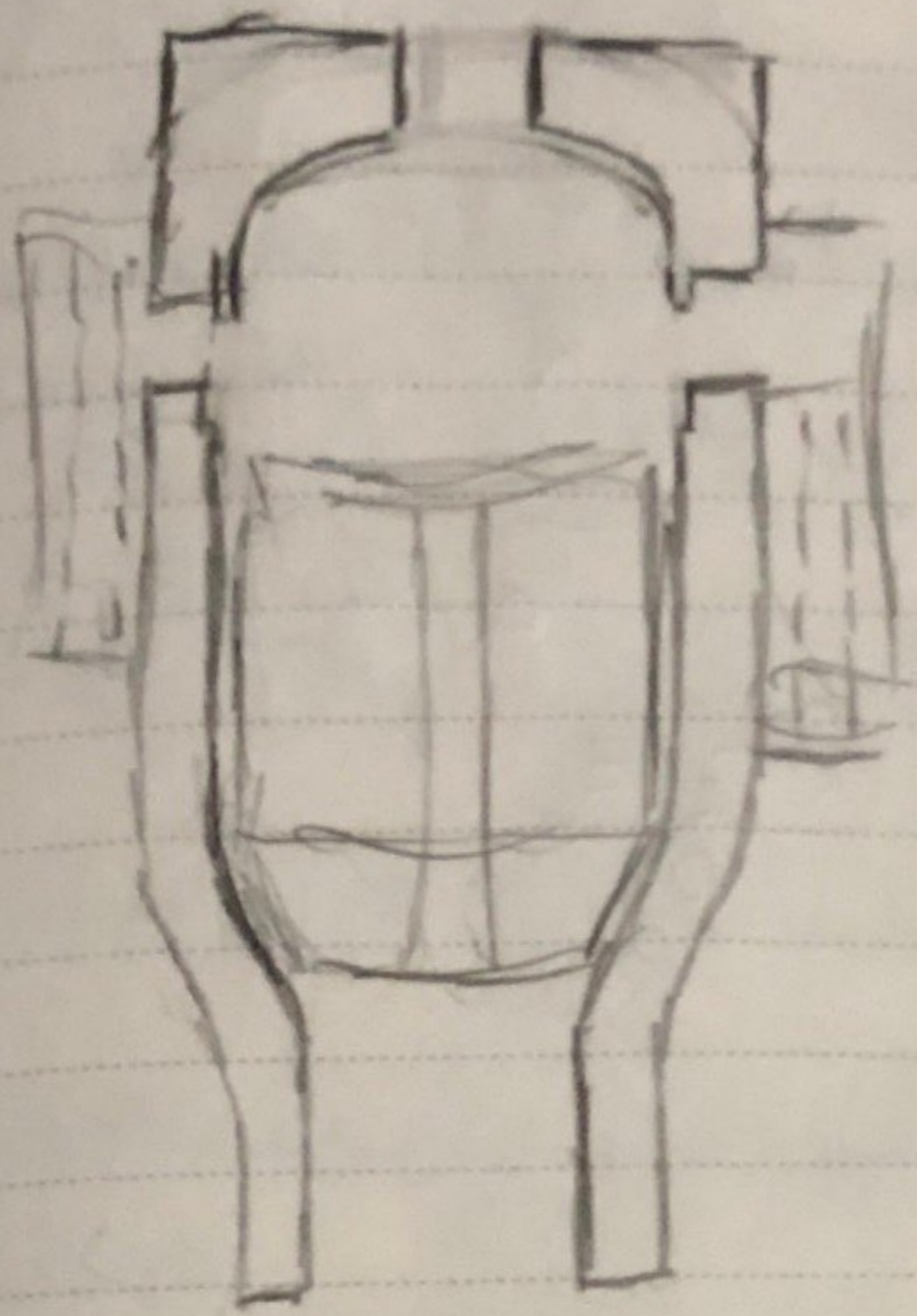
3D print

629

mark Forge



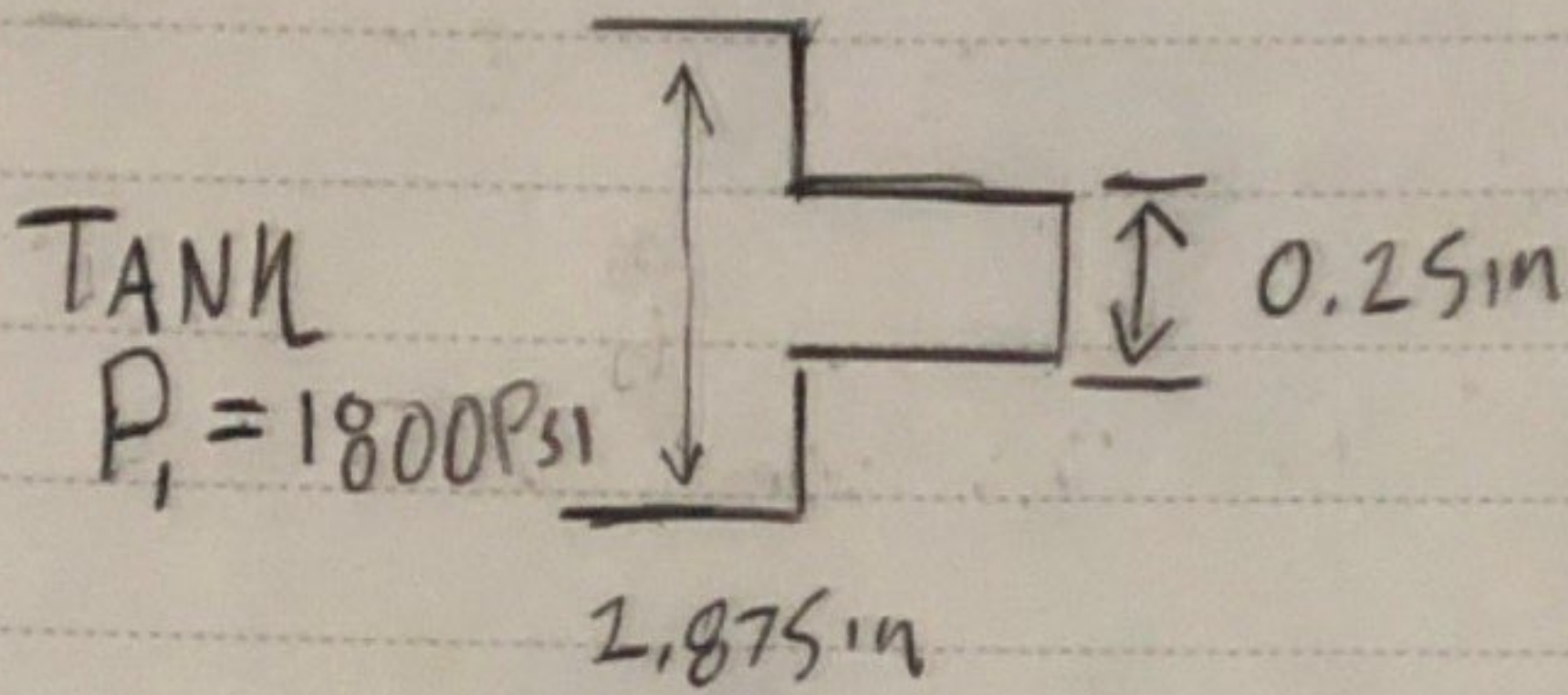




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# Pascal Principle

$$P_1 A_1 = P_2 A_2$$



Thoughts:  
Pascals principle, to verify,  
none overpressurization of the  
main tank.

Assumption - noncompressible  
fluid for ease of calculation

$$P_1 = 1800 \text{ PSI}$$

$$P_2 = ?$$

$$A_1 = 8.168 \text{ in}^2$$

$$A_2 = 0.0490 \text{ in}^2$$

walls 0.175 in

Tank  $\phi = 3.225 \text{ in}$

$$(1800 \text{ PSI})(8.168 \text{ in}^2) = P_2 (0.0490 \text{ in}^2)$$

$$300048.97 \text{ PSI} = P_2$$

$$P_{\text{max}} = 300.04 \text{ kpsi}$$

Chamber Inlet  
max Pressure

# Bolt Pattern

Date: \_\_\_\_\_

60° degrees  
Apart 4

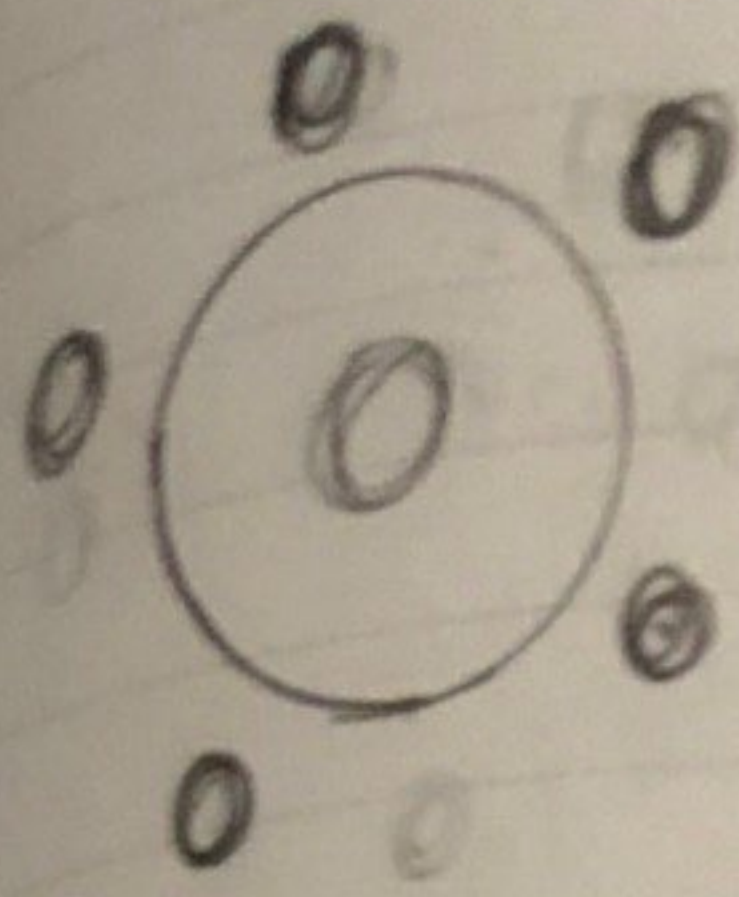
ALuminum  
26000psi

6061

180 MPa

$180 \times 10^6$  Pa

M4 x 0.7



$$F = 8 \times A$$

$$A = \frac{\pi}{4} (d - 0.9382 \times P)^2$$

$$A = \frac{\pi}{4} \times (4 - 0.9382 \times 0.7)^2$$

$d = 9 \text{ mm}$

$$\text{Pa} = \frac{\text{N}}{\text{m}^2}$$

$$A = 8.78 \text{ mm}^2$$

$$F = (180 \times 10^6 \frac{\text{N}}{\text{m}^2}) (8.78 \times 10^{-6} \text{ m}^2)$$

75% F.S.

$$F = 1,580.4 \text{ N} = 120.94 \text{ kg}$$

$$F = 1580.4 \text{ N}$$

$$F = 161.26 \text{ kg}$$

$$F_{\text{max}} = 7111.8 \text{ N} = 723.6 \text{ kg}$$

(All six with Preload)

$$F_{\text{max}} = 9482.4 \text{ N}$$

M5 x 0.8 mm  $d = 5 \text{ mm}$

$$A = \frac{\pi}{4} (d - 0.9382 \times P)^2$$

$$A = 14.1825 \text{ mm}^2$$

$$A = 1.4182 \times 10^{-2} \text{ m}^2$$

50% F.S.

$$F_{\text{max}} = 4741.2 \text{ N} = 483.79 \text{ kg}$$

$$F = 8 \times A$$

F.S. 0.75

$$F = (180 \cdot 10^6 \frac{\text{N}}{\text{m}^2}) (1.4182 \times 10^{-2} \text{ m}^2)$$

$$F_{\text{max}} = 5743.91 \text{ N}$$

$$F = 2552.85 \text{ N} \times 3$$

F.S. 0.91

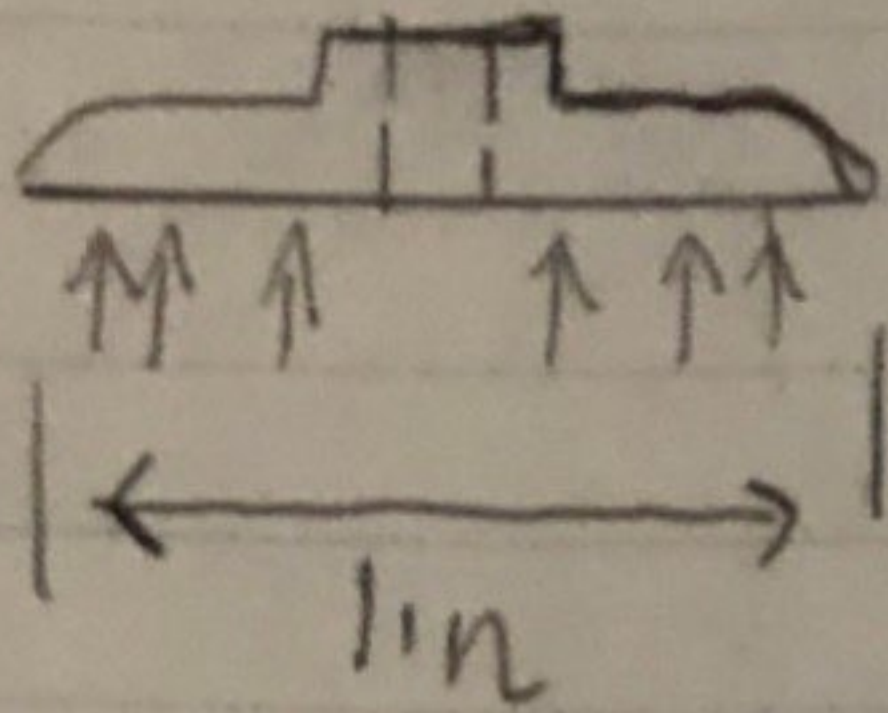
$$F_{\text{max}} = 7658.57 \text{ N}$$

$$F_{\text{max}} = 7000 \text{ N}$$



Date Force on cap

$$P_{max} = 300.04 \text{ ksi}$$



$$A = 0.000506 \text{ m}^2$$

$$A = 0.7854 \text{ in}^2 - 0.04909 \text{ in}^2 = 0.7363 \text{ in}^2$$

$$F = (P_{max})(A) = 220922.5 \text{ lbs}$$

~~$$F = 100419.29 \text{ kg}$$~~

$$P_{max} < 300.04 \text{ ksi}$$

In the event of stop in flow  
 $P_{max}$  is equal to max chamber pressure

f.s. 75%

$$F_{max} = 489.7 \text{ kg} = 1064.33 \text{ lbs}$$

$$F_{max} = 725.6 \text{ kg} = 1596.5 \text{ lbs}$$

$$1596.5 \text{ lbs} = (P_{max})(0.7363 \text{ in}^2)$$

$$(1064.33 \text{ lbs}) = (P_{max})(0.7363 \text{ in}^2)$$

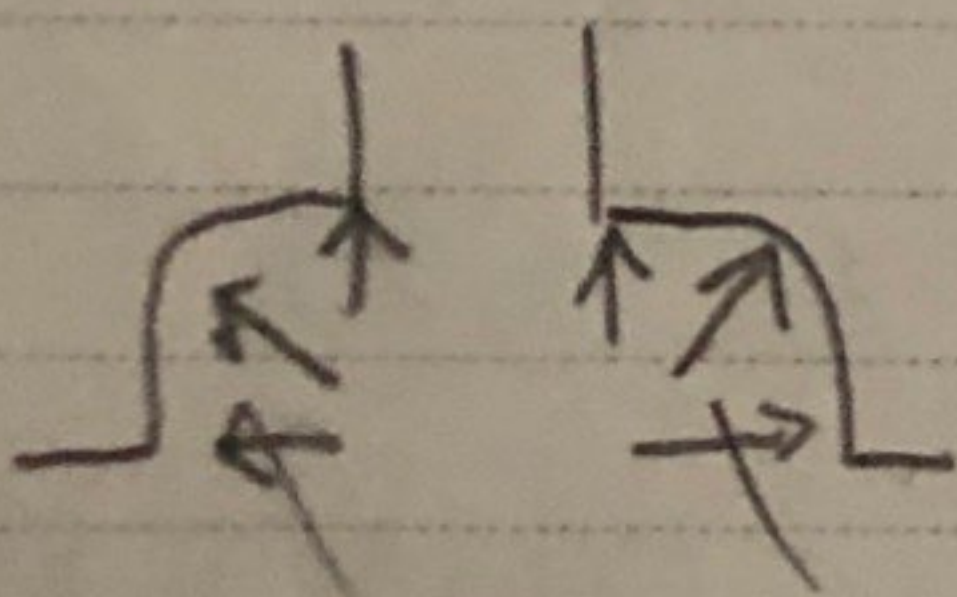
$$1445.52 \text{ Psi} = P_{max}^{90\%}$$

$$P_{max}^{75\%} = 2168.3 \text{ Psi}$$

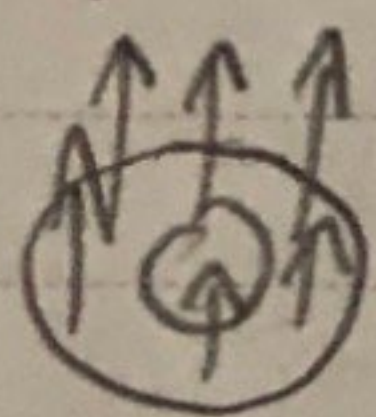
$$P_{max}^{75\%} \approx 2100 \text{ Psi}$$

$$P_{max}^{50\%} \approx 1500 \text{ Psi}$$

Still valid with hemisphere chamber.



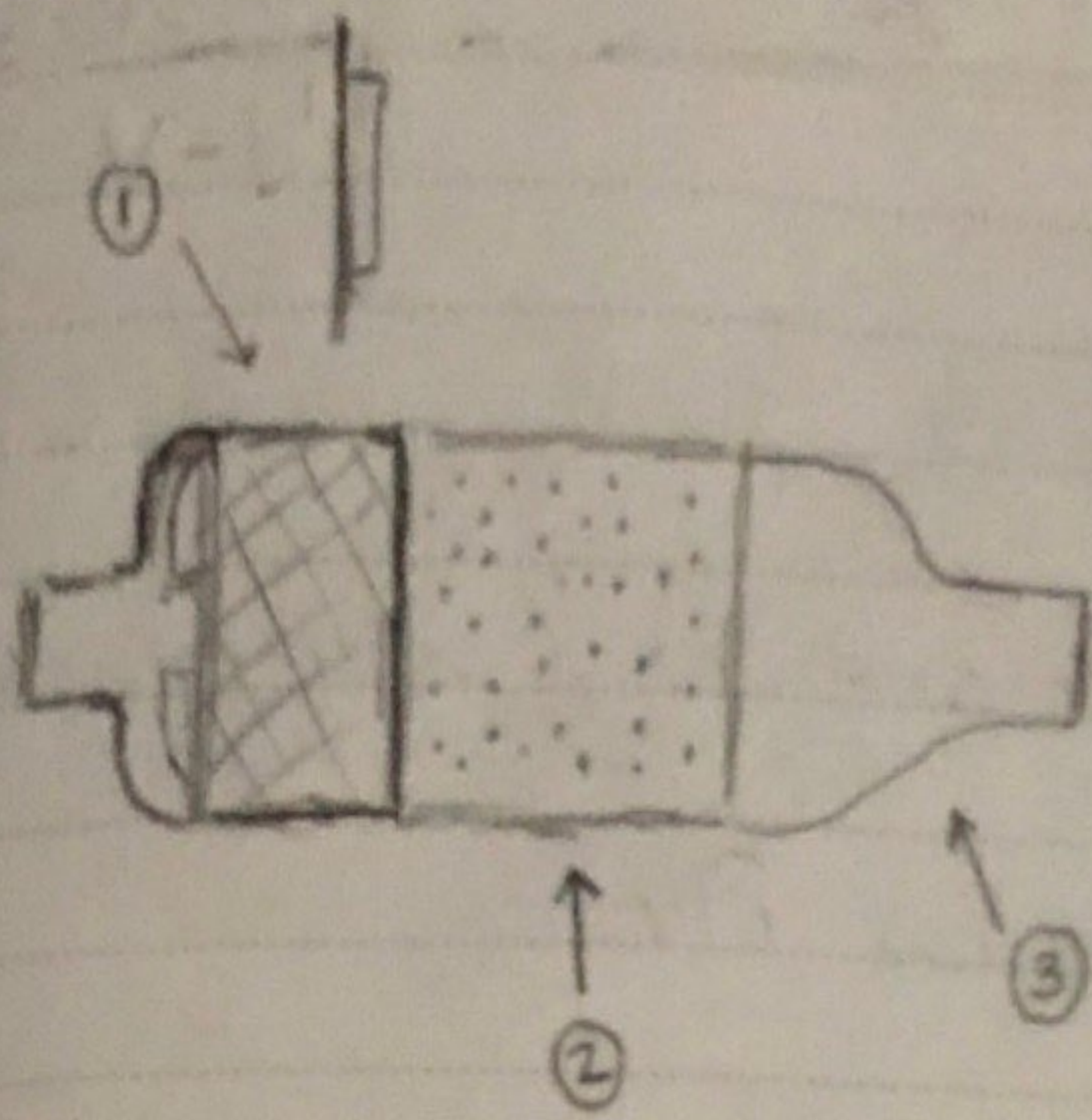
Opposing Forces cancel



the forces  $Z$  are the only ones left

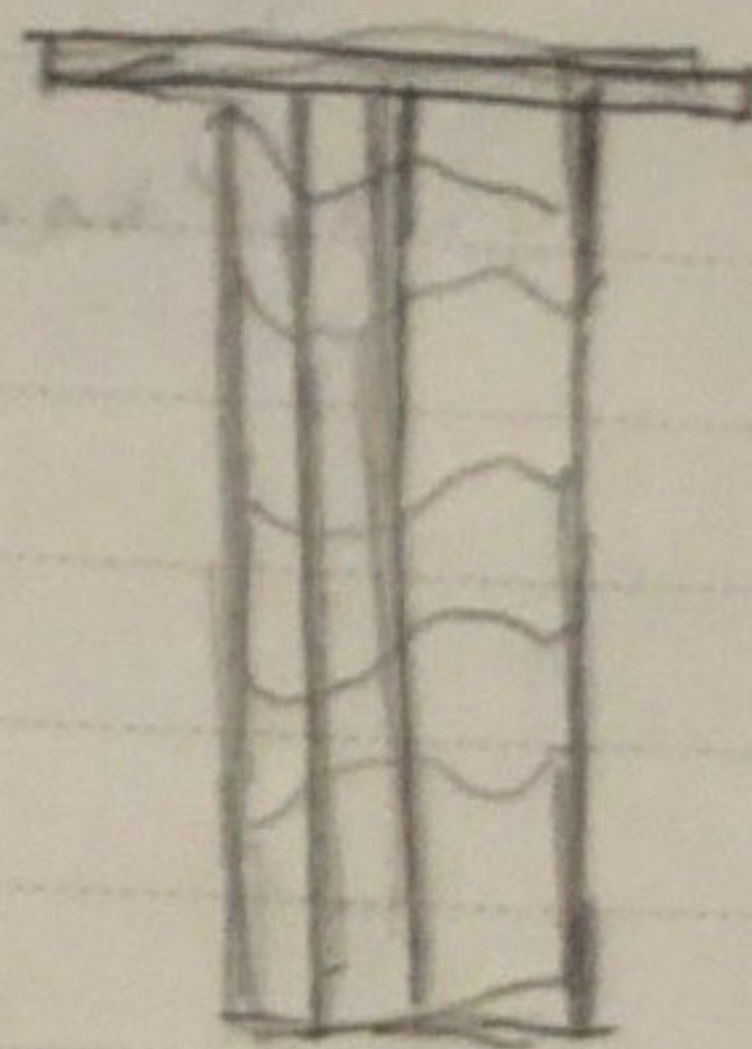
$$\sum F_x = 0$$

$$\sum F_y = P_{max}$$

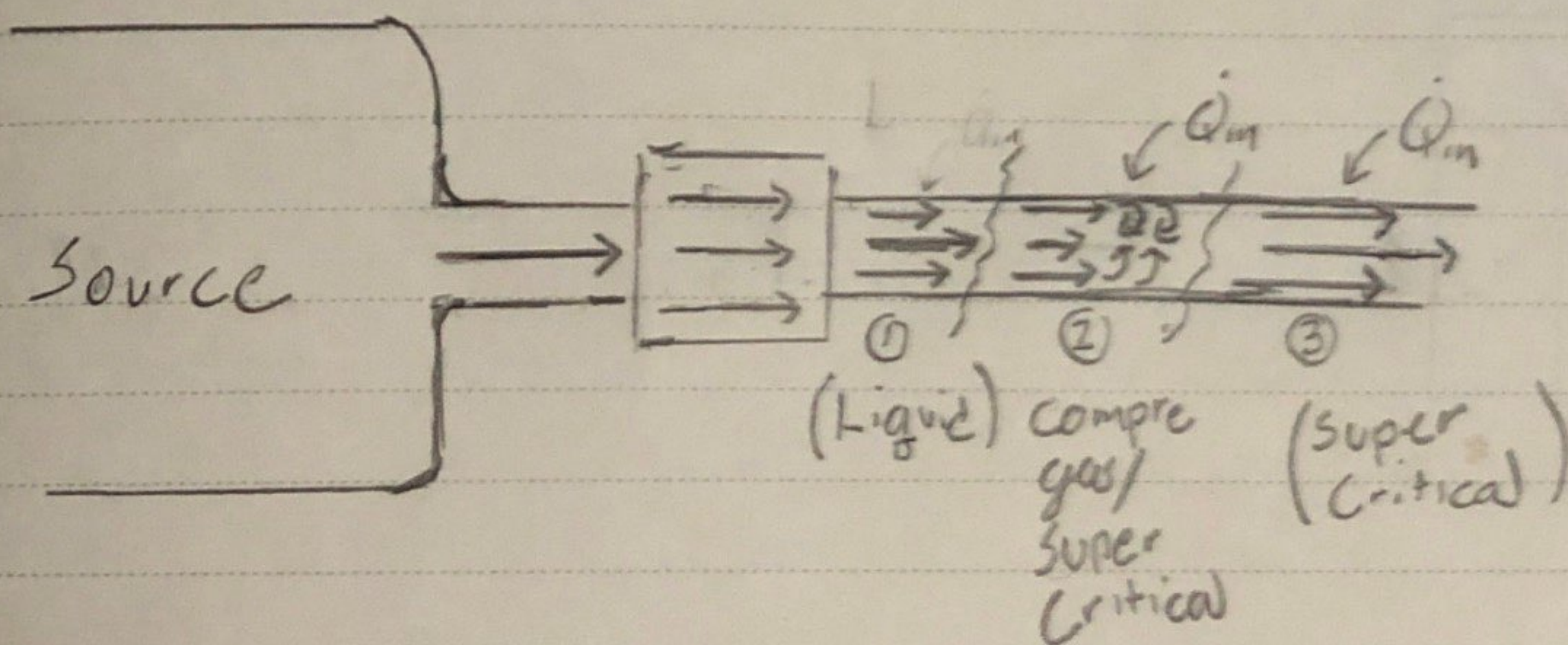


- 1) Diffusing chamber cap
- 2) Expansion chamber
- 3) Nozzel

Diffuser insert



### Theoretical Prediction



- 1) Liquid,  $CO_2$  lamnar Flow
- 2) Compressed gas / super critical turbulent Flow
- 3) super critical gas lamnar Flow

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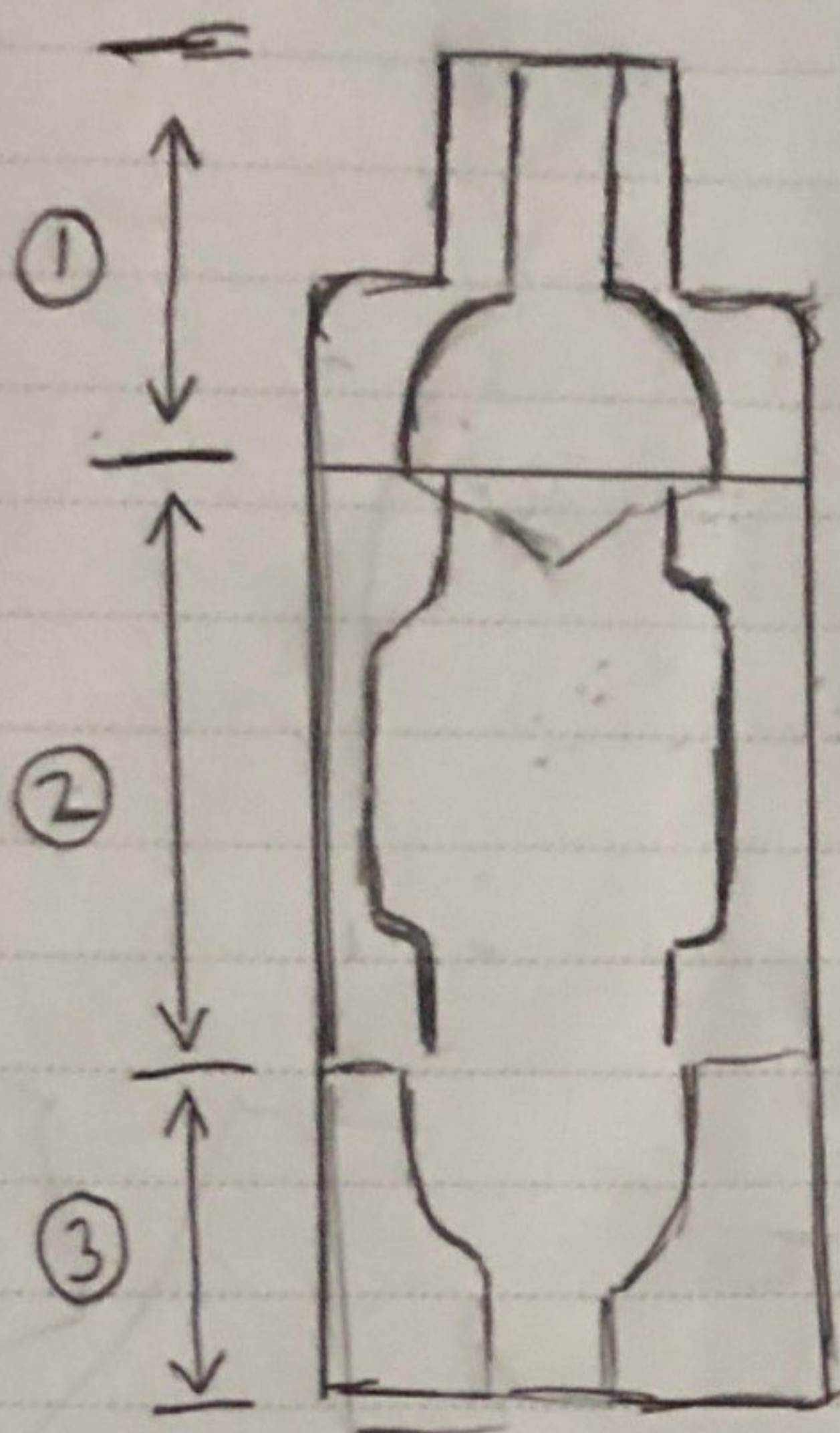
# Phase Diagram

1) liquid Phase near  $C_p @ 9000$

2) supercritical / Compressible liquid Phase  
@  $T = 50-75^\circ C$

3) gaseous Phase

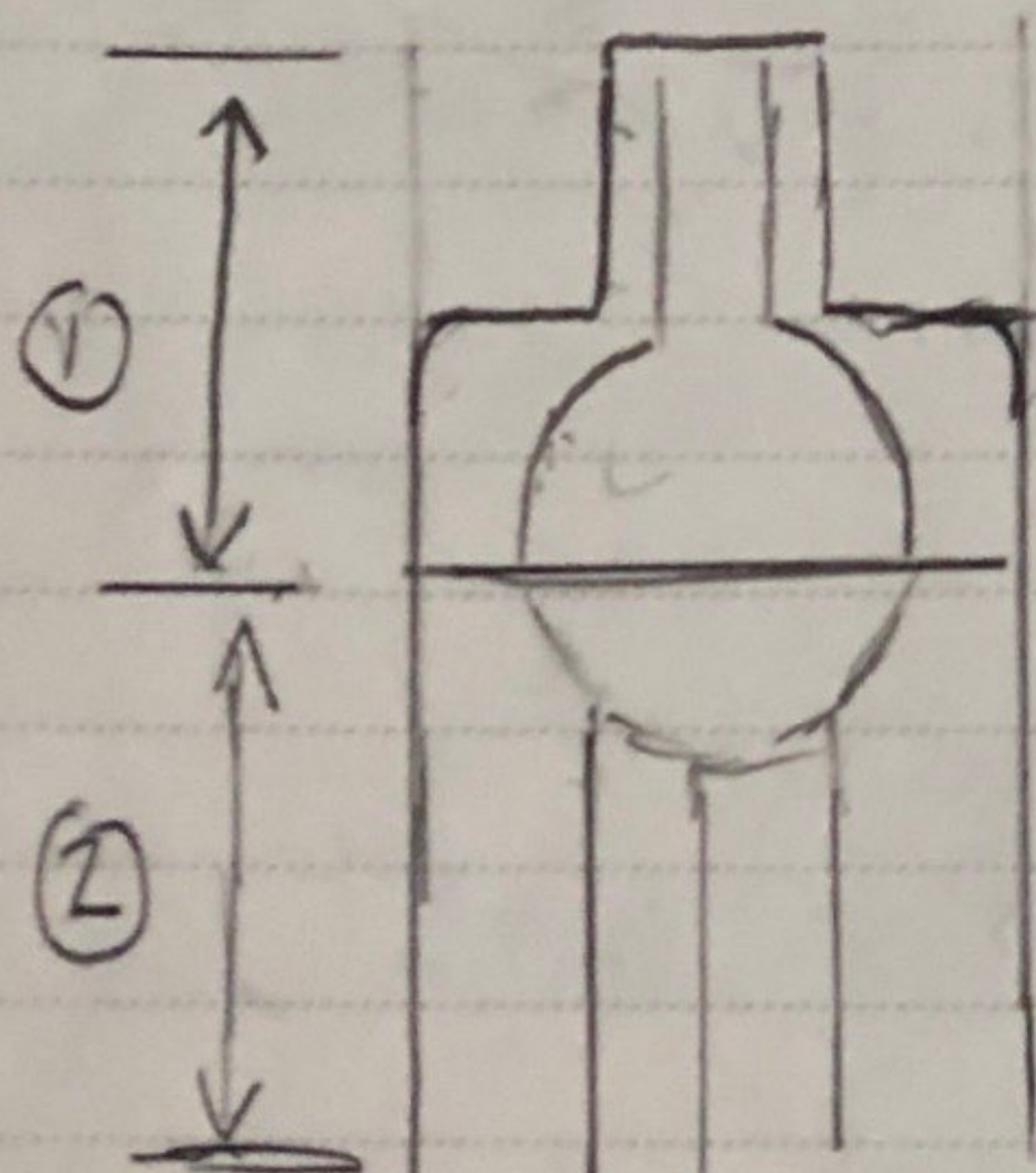
(Inductive)



Inductive Heater  $\dot{Q} = 500 \text{ w } \frac{1}{s}$

Resistance Heater  $\dot{Q} = 180 \text{ w } \text{ or } \frac{1}{s}$

(Resistance)



(R K E)

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$$a' = \left( \frac{0.4278}{Z_c} \right) R V_c T_c^{3/2}$$

$$P = \frac{RT}{V-b'} - \frac{a'}{\sqrt{TV}(V+b')}$$

$$b' = \left( \frac{0.0867}{Z_c} \right) V_c$$

$$V_c = \frac{RT_c Z_c}{P_c}$$

$V$  = molar volume  $\frac{\text{Vol}}{\text{mol}}$

$P$  = Pressure

$R$  = gas constant

$T_c, Z_c, P_c$  } Table

(Energy Equation)

Tank Volume  
 $52.11 \text{ in}^3 = 0.000853 \text{ m}^3$   
 $P = 900 \text{ PSI} = 62 \text{ bar}$   
 $6205500 \text{ Pa}$

$\Sigma E$  (closed system,  $\Delta M = 0$ )

$$(W_{in} - W_{out}) + (Q_{in} - Q_{out}) = \Delta U + \Delta KE + \Delta PE$$

$$Pa = \frac{N}{m^2}$$

$$N = kg \frac{m}{s^2}$$

$$\Sigma E = \Delta KE + \Delta PE$$

Tank P.E.

$$\Delta PE = P(V)$$

$$\Delta PE = (6205500 \text{ (N)}/m^2) (0.000853 \text{ m}^3)$$

$$\Delta PE = 5293.29 \text{ N}\cdot\text{m}$$

Body Falling

$$\Delta KE = (5 \text{ kg})(9.8 \text{ m/s}^2)(3 \text{ m}) + \frac{1}{2} (15 \text{ kg})(5 \text{ m/s})^2$$

$$\Delta KE = 147 \text{ kg} \frac{m^2}{s^2} + 62.5 \text{ kg} \frac{m^2}{s^2}$$

$$\Delta KE = 209.5 \text{ N}\cdot\text{m}$$

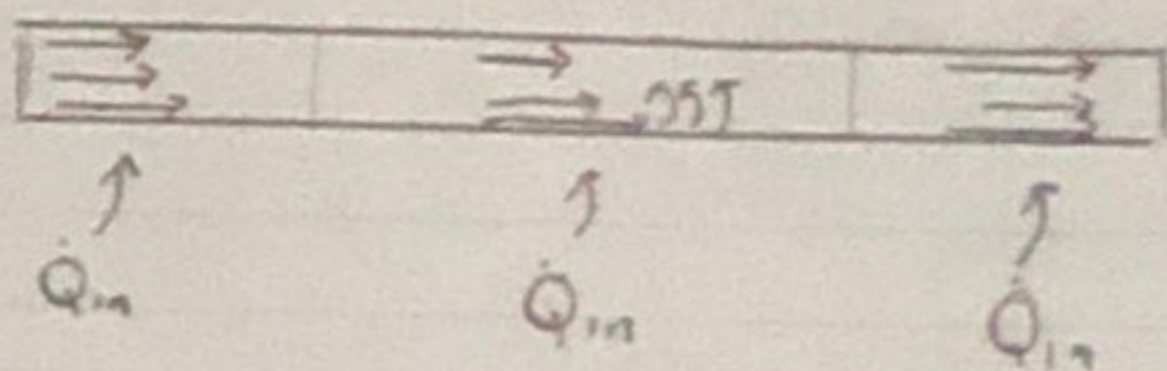
$$\Delta PE_{\text{Tank}} > \Delta KE_{\text{Body}}$$

$$\phi = 2.98 \times 10^{-3} \text{ gm}$$

0.16 m

# Theoretical Prediction

Date: \_\_\_\_\_



$$\dot{m} = \frac{0.1469 \text{ kg/s}}{6}$$

$$\dot{m} = 0.0244 \text{ kg/s}$$

Heat eng  
 $\dot{Q}_{in} = m C \Delta t$   
 ↑  
 mass

heat capacity

$$20^\circ\text{C} = 293.15 \text{ K} \quad \dot{Q}_{in} = \frac{500 \text{ J/s}}{6}$$

$$\text{mol mass} = 12.01 \text{ g/mol} \quad \dot{Q}_{in} = 80 \text{ J/s}$$

$$0.01201 \text{ kg/mol} \quad C_v = 28.96 \text{ J/mol}\cdot\text{K}$$

$$80 \text{ J/s} = (0.0244 \text{ kg/s}) (28.96 \text{ J/mol}\cdot\text{K}) (T_f - 293.15 \text{ K})$$

$$80 \text{ J/s} = (2.0316 \text{ mol/s}) (28.96 \text{ J/mol}\cdot\text{K}) (T_f - 293.15 \text{ K})$$

$$80 \text{ J/s} = (58.836 \text{ J/s}\cdot\text{K}) (T_f - 293.15 \text{ K})$$

$$1.357 \text{ K} + 293.15 \text{ K} = T_f$$

$$294.507 \text{ K} = T_f$$

1000 watts

160 J/s

$$160 \text{ J/s} = (58.836 \text{ J/s}\cdot\text{K}) (T_f - 293.15 \text{ K})$$

$$2.719 + 293.15 \text{ K} = T_f$$

$$295.869 \text{ K} = T_f$$

$$\Delta T = (150^\circ\text{C} - 20^\circ\text{C})$$

$$50^\circ\text{C} = 323.15 \text{ K}$$

$$\frac{\dot{Q}_{in}}{6} = (2.0316 \text{ mol/s}) (28.96 \text{ J/mol}\cdot\text{K}) (323.15 - 293.15)$$

$$(2.0316 \text{ mol/s}) (28.96 \text{ J/mol}\cdot\text{K}) (29.65 \text{ K})$$

$$\dot{Q}_{in} = 10966.8 \text{ J/s}$$

180 watts

$$\dot{m} = 0.1469 \text{ kg/s}$$

$$\dot{m} = 12.2314 \text{ mol/s}$$

$$\text{mol mass} = 0.01201 \text{ kg/mol}$$

$$\dot{Q}_{in} = \dot{m} (C) (\Delta T)$$

$$180 \text{ J/s} = (12.231 \text{ mol/s}) (28.96 \text{ J/mol}\cdot\text{K}) (\Delta T)$$

$$0.508 \text{ K} = \Delta T$$

2500 watts

$$2500 \text{ J/s} = (12.231 \text{ mol/s}) (28.96 \text{ J/mol}\cdot\text{K}) (\Delta T)$$

$$7.05 \text{ K} = \Delta T$$

1/4 in

$$\Delta T = 28.23$$

$$\dot{m} = \rho A v$$

$$0.0367 \frac{\text{kg}}{\text{s}} = (1.97 \text{ kg/m}^3) A (2481.74 \text{ m/s})$$

$$7.9134 \times 10^{-6} = A$$

$$\boxed{D = 0.125}$$

①  $1/4$  opening

$$\dot{m} = 0.1469 \text{ kg/s}$$

$$V = 2481.74 \text{ m/s}$$

$$\rho = 131.6 \text{ kg/m}^3$$

$3/16$  opening

$$0.1875 \text{ in} = 0.004762 \text{ m}$$

$$A = 0.00001781 \text{ m}^2$$

$1/8$  opening

$$0.125 \text{ in} = 0.003175 \text{ m}$$

$$A = 0.000007917 \text{ m}^2$$

$$\dot{m} = \rho A V$$

$3/16$

$$\dot{m} = \left( 131.6 \frac{\text{kg}}{\text{m}^3} \right) \left( 0.00001781 \text{ m}^2 \right) \left( 2481.74 \text{ m/s} \right)$$

$$\dot{m} = 5.816 \text{ kg/s}$$



# CO<sub>2</sub> cooling loop

