Notes / Assumption

$Pa = \frac{N}{m^2}$ $N = kg\frac{m}{s^2}$

Tank @ 50°C or 323.15 K

$C_{v_{CO2}} = 28.96\ J/mol\cdot k$

Supercritical state @ 73.8 bar, 1070.37 Psi
Tank 7.38 Mpa

Incompressible Fluid

Tank Volume 52.11 in^2 = 0.000853 m^3

$\rho = 200\text{-}400 \frac{kg}{m^3}$ @ 75 bar

mol mass = 0.01201 kg/mol

Redlich Kwong Eq of State

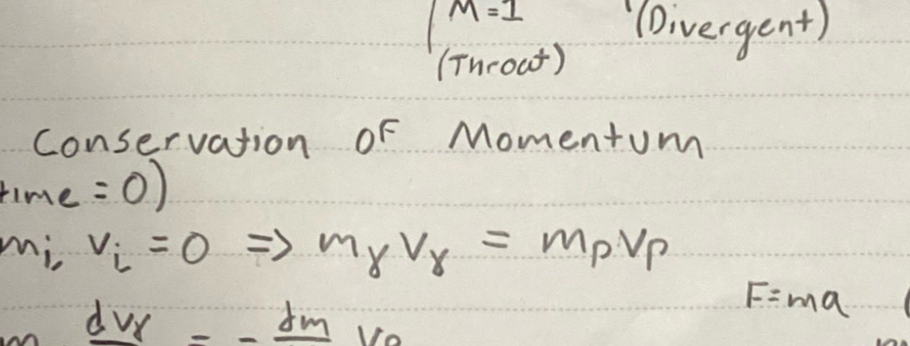
$$P = \frac{RT}{V-b'} - \frac{a'}{(\sqrt{T})\,V(V+b')} \qquad V_c = \frac{RT_cZ_c}{P_c}$$

V = molar
P = Pressure
T = Temp
R = gas constant

$$a' = \left(\frac{0.4278}{Z_c}\right)RV_cT_c^{3/2} \qquad b' = \left(\frac{0.0867}{Z_c}\right)V_c$$

T_c, Z_c, P_c } All tables

Rocket Nozzle



Specific impulse
(Efficiency)

$$I_s = V_2/g$$

$$a = \sqrt{\frac{\gamma\cdot R\cdot T}{M}}$$

$$V_e = \sqrt{\frac{T\cdot R}{M}\cdot\frac{2\gamma}{\gamma-1}\left[1-\left(\frac{P_a}{P}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

Conservation of Momentum
(time = 0)

$$m_i v_i = 0 \Rightarrow m_\gamma V_\gamma = m_p V_p$$

$$m_\gamma \underbrace{\frac{dv_\gamma}{\partial t}}_{\text{accelertion}} = -\underbrace{\frac{\partial m}{\partial t}}_{\dot{m}} V_p$$

(note)

$$F = \dot{m}V_2 + (P_2 - P_3)A_2$$

F=ma (Ideal Nozzel Expansion)

$$m_\gamma a_\gamma = -\dot{m}\ V_p$$

$$F = \dot{m}V_p$$

Date:

(Diagram)

(System Energy Equation)

ΣE (closed system, $\overset{\Delta Q = 0}{\Delta M = 0}$)

Tank Volume

$V = 0.000853 \, m^3$

$$(\cancelto{0}{W_{in} - W_{out}}) + (\cancelto{0}{Q_{in} - Q_{out}}) = \cancelto{0}{\Delta U} + \Delta KE + \Delta PE$$

$P = 7.38 \times 10^6 \, Pa$

$Pa = \frac{N}{m^2}$

$N = kg \frac{m}{s^2}$

$$\Sigma E = \Delta KE + \Delta PE$$

(TANK P.E.)

$$\Delta PE = P(V)$$

$$\Delta PE = (7.38 \times 10^6)(N/m^2)(0.000853)(m^3)$$

$$\Delta PE = 6295.14 \, N \cdot m$$

(Body Falling)

$$\Delta KE = (5kg)(9.8 \, m/s^2)(3m) + \frac{1}{2}(5kg)(5\frac{m}{s})$$

$$= 147 \, kg \, m^2/s^2 + 62.5 \, kg \, m^2/s^2$$

$$\Delta KE = 209.5 \, N \, m$$

$$\Delta PE_{Tank} > \Delta KE_{Body}$$

(Bernoulli's Equation)

$\Delta P = 1070 Psi - 14.7 PSI$

$= 1055.3 Psi$

$= 7.38 * 10^6 Pa$

$$V = \sqrt{\frac{2\Delta P}{\rho}}$$

$$V : \boxed{135.83 m/s}$$

$\rho = 800 kg/m^3$

$V = m/s$

$V = 135.83 m/s$

$\rho = 800 kg/m^3$

20oz = 0.5669 kg

Ø = 0.25 = 6.37 mm

$A = 0.00003167 m^2$

(Mass Flow)

From TANK

$$\dot{m} = \rho A V$$

3/16 = 0.004762 m

$A = 0.00001781 m^2$

Ø = 0.25

1/8 = 0.003175 m

$A = 0.000007917 m^2$

$$\dot{m} = (400 kg/m^3)(0.00003167 m^2)(135.83 \tfrac{m}{s})$$

$$\dot{m} = 1.720 kg/s$$

Time

$$\frac{m}{\dot{m}} = t$$

$$\frac{0.5669 kg}{1.720}\left(\frac{s}{kg}\right) = 0.3294 s$$

$\varnothing = 3/16$

$$\dot{m} = (1400\ kg/m^3)(0.00001781\ m^2)(135.83\ m/s)$$

$$\dot{m} = 0.9676\ kg/s$$

Time

$$\frac{0.5669\ kg}{0.9676}\left(\frac{s}{kg}\right) = 0.5858\ s$$

$\varnothing = 1/8$

$$\dot{m} = (1400\ kg/m^3)(0.000007917\ m^2)(135.83\ m/s)$$

$$\dot{m} = 0.4301\ kg/s$$

$$\frac{0.5669\ kg}{0.4301}\ \frac{s}{kg} = 1.3180\ s$$

$\varnothing_{Act} = 0.340\ in$ $\qquad$ $A = 0.00005858\ m^2$

$$\dot{m} = (1400\ kg/m^3)(0.00005858\ m^2)(135.83\ m/s)$$

$$\dot{m} = 3.1827\ kg/s$$

(Flow is divid into 3 Flow Paths)

$\dot{m}_{FG}$ $\quad \underline{\dot{m}_{total}}$ $\qquad$ $\dot{m}_{told} = \dot{m}_1 + \dot{m}_2 + \dot{m}_3$

(Mass Flow)

Frist Face $A_{F1} = 0.00003365 m^2$ $\dot{m}_{FF} = 1.0609 \, kg/s$

$$\dot{m}_{FF} = \rho A V$$

$$1.0609 \, kg/s = (400 \, kg/m^3)(0.00003365 \, m^2)(V)$$

$$V_{FF} = 78.818 \, m/s$$

Heater Tube

Divid by two Flow

$$A = 0.00001140 \, m^2$$

$$\dot{m}_{HT} = \frac{\dot{m}_{FF}}{2} = 0.5304$$

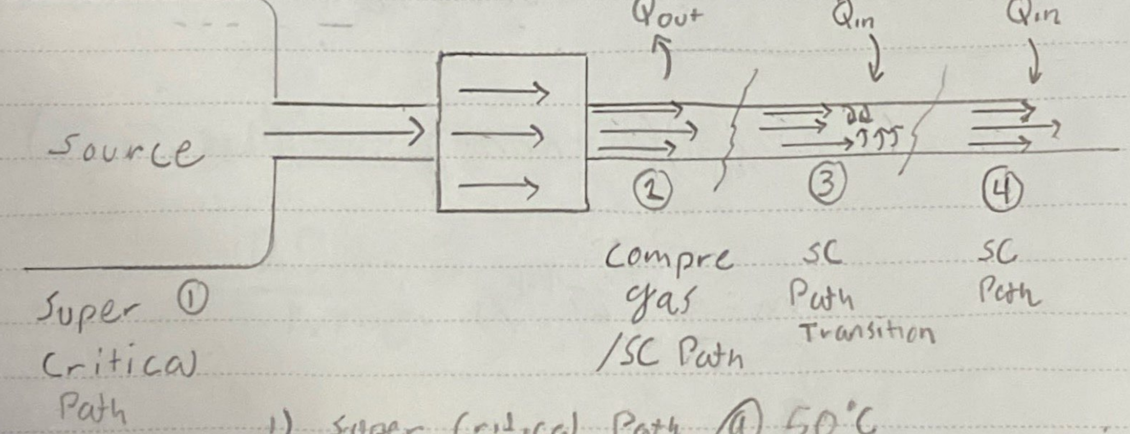
$$\dot{m}_{HT} = \rho A V$$

$$\dot{m}_{HT} = \dot{m}_1 + \dot{m}_2$$

$$0.5304 \, \frac{kg}{s} = (400 \, kg/m^2)(0.00001140 m^2) \, V$$

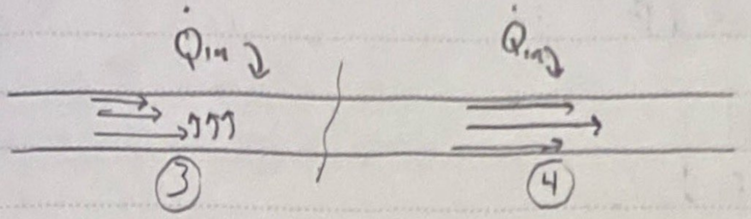
$$116.326 \, m/s = V$$

Theoretical Prediction



1) Super Critical Path @ 50°C
2) Compre/SC Path Because of Cooling Q_{out}
3) Transition Zone SC $\dot{Q}_{in}$
4) SC Flow

Theoretical Prediction



(CO$_2$)

$C_v = 28.96\ \text{J/mol·K}$

mol mass $= 0.01201\ \text{kg/mol}$

$\dot{Q}_{total} = 2500\ \text{J/s}$

$\dot{m}_{HT} = 0.5304\ \text{kg/s}$

Heat energy

heat capacity

$$\dot{Q}_{in} = \dot{m}\, c\, \Delta t$$

mass Flow

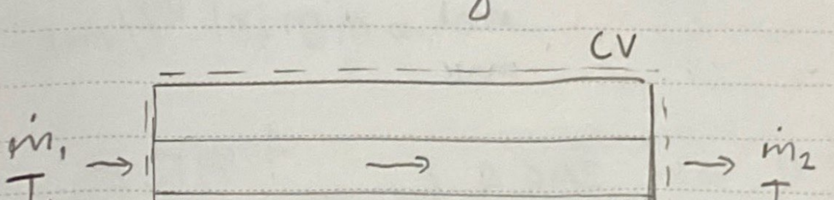
$$\dot{Q}_{per\ path} = \frac{\dot{Q}_{total}}{6} = \frac{2500\ \text{J/s}}{6} = 416.6\ \text{J/s}$$

$$2500\ \text{J/s} = 44.163\ \text{mol/s}\ (28.96\ \text{J/mol·K})\ \Delta t$$

$$\boxed{1.95\text{K} = \Delta t}$$

$H = H'(T) + \int (C_p\, dT) - (PV)$

Heat Exchanger mobel



STEADY-STATE

$\dot{m}_1 = \dot{m}_2$

$\dot{m}_1 = 0.5304\ \text{kg/s}$

$T_1 = 325.15\ \text{K}$

$P_1 = 7.38\ \text{MPa}$

$\dot{Q}_{in} = 416.6\ \text{J/s}$

$$\frac{dE}{dt} = 0 = \dot{Q}_{in} - \cancelto{0}{\dot{W}} + \dot{m}_1\left(h_1 + \cancelto{0}{\frac{V_1^2}{2}} + \cancelto{0}{gz_1}\right) - \dot{m}_2\left(h_2 + \cancelto{0}{\frac{V_1^2}{2}} + \cancelto{0}{gz_1}\right)$$

$$0 = \dot{Q}_{in} + \dot{m}_1(h_1) - \dot{m}_2(h_2)$$

H Enthalpy : 472800 J/kg

$$416.6\ \text{J/s} = 0.5304\ \text{kg/s} \qquad \dot{Q}_{in} = -\dot{m}(h_1 - h_2)$$

Enthalpy : 1863 J/kg·K

$$-785.444\ \text{J/kg} = 472800\ \text{J/kg} - h_2$$

$h_2 = 473585.4441\ \text{J/kg}$

Date:

$$dH = T\,ds + V\,dP$$

$$\left.\frac{\partial H}{\partial s}\right|_P = T \qquad \left.\frac{\partial H}{\partial P}\right|_s = V$$

(Maxwell Relations)

$$\left.\frac{\partial T}{\partial P}\right|_s = \left.\frac{\partial V}{\partial s}\right|_P$$

Fudamental Thermodynami
Relationships

$$dU = T\,ds - P\,dV$$
$$dH = T\,ds + V\,dP$$

$f(x,y)$

$$dF = \left.\frac{\partial f}{\partial x}\right|_y dx + \left.\frac{\partial F}{\partial y}\right|_x dy$$

Thermodynamic Quantitie

$$C_V = \left.\frac{\partial U}{\partial T}\right|_V$$

Heat Exchanger model

$K_2 = 473585.444$ J/kg × $\frac{\text{mol}}{\text{mass}}$ $\dot{m} = 0.5304$ kg/s

$$\Delta T = \frac{5687.761 \text{ J/mol}}{C_V} \quad \frac{\text{mol}\cdot\text{K}}{\text{J}}$$

$C_V = 28.96$ J/mol·K

$\Delta T = 196.40$ K

$\frac{\text{mol}}{\text{mass}} = 0.01201$ kg/mol

$\underline{T_2 = 521.55 \text{ K}}$ or 248.4 C, 479.12 F

(RK Eq of State)

$$P = \frac{RT}{V - b'} - \frac{a'}{(\sqrt{T})V(V+b')}$$

Found by simulation

$$V_c = \frac{R T_c Z_c}{P_c}$$

$$a' = \left(\frac{0.4278}{Z_c}\right) R V_c T_c^{3/2}$$

$$b' = \left(\frac{0.0867}{Z_c}\right) V_c$$

$\dot{m}_2 = 0.5304 \frac{kg}{s}$

$T_2 = 521.55k$

Proof Converging Diverging Nozzel

$$V_e = \sqrt{\frac{T \cdot R}{M} \cdot \frac{2 \cdot \gamma}{\gamma - 1} \cdot \left[1 - \left(\frac{P_e}{P}\right)^{\frac{\gamma - 1}{\gamma}}\right]}$$

$T = K°$

$R = 8314.5 \, \frac{J}{kmol \, K}$

$$a = \sqrt{\frac{\gamma \cdot R \cdot T}{M}}$$

M = molecular mass g/mol

$\gamma = 1.289$

$P = 100$ bar

$P_e = 1.01325$ bar

γ = Ratio of specific heat

$T = 287.56K$

$M = 44.01 \, g/mol$

P_e = atm

P = Stating Pressure

$$V_e = \sqrt{(54326.689)(9.0175)(0.6388)}$$

$$V_e = \sqrt{312966.60}$$

$$V_e = 559.412$$

$$a = 264.21$$

$$V_e > a$$

Throat

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

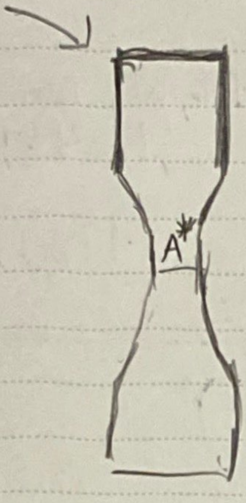
M = Mach #

A^* = Small Point

A = Aera of intre

$V_1 = 89.39$ m/s

$A = 0.0000334\ m^2$



$$M_1 = \frac{89.39}{264.21}$$

$$M_1 = 0.3383$$

$$M = \frac{V}{a}$$

$\gamma = 1.285$

$$\frac{0.0000334\ m^2}{A^*} = \frac{1}{0.3383}\left[\frac{2}{2.285}\left(1+\frac{0.285}{2}(0.3383^2)\right)\right]^{4.0087}$$

$$0.0000334 m^2 = A^* 2.7606$$

$$1.2098 \times 10^{-5} = A^*$$

$$0.000012098 m^2 = A^*$$

$\gamma = 1.9624$ mm

$A = \pi r^2$

$\gamma = 1.9624 \times 10^{-3}$

$\varnothing = 3.9248 \times 10^{-3}$ m

$V_e = 559.412$

$a = 264.21$

0.002,778 m

$$M = \frac{V}{a} = 2.117$$

$$\frac{A}{0.000012098 m^2} = \frac{1}{2.117}\left[\frac{2}{2.285}\left(1+\frac{0.285}{2}(2.117)^2\right)\right]^{4.0087}$$

$$\frac{A}{0.000012098} = 2.00517 = 2.4258 \times 10^{-5}\ m^2$$

$\gamma = 2.778 \times 10^{-3}$ m

$\varnothing = 5.556 \times 10^{-3}$ m

12 Date: Bolt Pattern

Markforged
Ultimate Tensile Strength
Inconel 625
765 Mpa at 25°C
558 Mpa at 600°C

M5 x 0.8 mm   Ø = 5mm

$A = \frac{\pi}{4}(d - 0.9382 \cdot P)^2$

$A = 1.4182 \times 10^{-5}\ m^2$

(625) @ 300°C

$F_{max} = \gamma A$

Ultimate Tensile Strength
17-4 PH v2

1180 Mpa

$F_{max} = (661.5 \times 10^6\ N/m^2)(1.4182 \times 10^{-5}\ m^2)$

$F_{max} = 9381.39\ N$

F.S. = 0.75

$F_{0.75} = 7036.04\ N$

×3

$F_{0.75_{total}} = 21108.13\ N$

(17-4 PH)

$F_{max} = (1180 \times 10^6\ N/m^2)(1.4182 \times 10^{-5}\ m^2)$

$F_{max} = 16734.76\ N$

$F_{0.75} = 12551.07\ N$

×3

$F_{0.75_{total}} = 37653.21\ N$

$F_{0.5} = 8367.38\ N$

×3

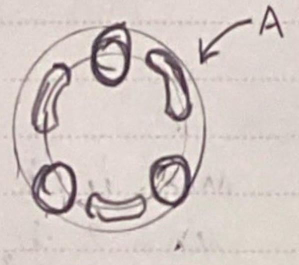
$F_{0.5_{total}} = 25102.14\ N$

Bolt Pattern

$Pa = \frac{N}{m^2}$

$283 \text{ bar} = 283,00000 \text{ Pa}$

$A = 0.00004145 \text{ m}^2$



$F_p = PA$

$F_p = 1173.035 \text{ N}$

$\times 3$

$F_{p_{total}} = 3519.105 \text{ N}$

$F_{total} < F_{p_{total}}$

Bolt pattern is safe

Date: Thrust

(Sim)

Velocity: Max $141.04\,\frac{m}{s}$

Min $65.34\,\frac{m}{s}$

$$F_T = \dot{m}_e V_e + (P_e - P_0) A_e$$

Mach #

Min 1.38

Max 2.86

$$(2.018\,\tfrac{kg}{s})(141.04\,\tfrac{m}{s}) + (5205525\,\tfrac{N}{m^2})\,A$$

$$284.61\,N + 133.365\,N$$

$$F_T = 417.983\,N = 42.65\,kg$$

$\dot{m} = \rho V A$

$A = 0.00002562\,m^2$

$\rho = 558.57\,\frac{kg}{m^3}$

$\dot{m}_e = 2.018\,\frac{kg}{s}$

$P_0 = 1\,atm = 101325\,\frac{N}{m^2}$

$P_e = 53.068\,bar = 5306850\,N/$